Homework Set on
Probabilistic Failure Models

List of problems:

[0] \lectures\reltest\pfm_ps.tex 29.5.2016

1. **Time to failure: 1.** (p.14) A component with time to failure T has constant failure rate:

$$z(t) = \lambda = 2.5 \times 10^{-5} \text{ [hours]}^{-1} \tag{1}$$

   (a) Determine the probability that the component survives a period of 2 months without failure.

   (b) Find the mean time to failure (MTTF) of the component.

   (c) Find the probability that the component survives its MTTF.

2. **Time to failure: 2.** (p.14) A machine with constant failure rate λ will survive a period of 100 hours without failure, with probability 0.50.

   (a) Determine the failure rate λ.

   (b) Find the probability that the machine will survive 500 hours without failure.

   (c) Determine the probability that the machine will fail within 1000 hours, when you know that the machine was functioning at 500 hours.

3. **Time to failure: 3.** (p.15) A component with time to failure T has failure rate:

$$z(t) = kt \quad \text{for} \quad t > 0 \quad \text{and} \quad k = 2.0 \times 10^{-6}[\text{hours}]^{-2} \tag{2}$$

   (a) Determine the probability that the component survives 200 hours.

   (b) Determine the mean time to failure, MTTF, of the component.

   (c) Determine the probability that a component, which is functioning after 200 hours, is still functioning after 400 hours.

4. **Scratches.** (p.16) (Poisson and exponential distributions). Scratches are introduced randomly and independently along magnetic tape as it is produced by a particular procedure. The average density of scratches on the tape is $\lambda = 0.003$ scratches per meter.

   (a) What is the probability that a 200 meter segment is free of scratches?

   (b) 1000 meter tapes are "acceptable" if the number of scratches is no more than 2. In a box of 50 tapes, what is the probability that all tapes are acceptable?

   (c) What is the mean scratch-free length?

5. **Many identical components.** (p.17) (Weibull distribution). A device has a large number of identical components. The device fails as soon as the first component fails. The failure rate function for this device increases linearly in time, and the mean time to failure is 125 hours. What is the probability that the device will operate at least 175 hours?

6. **Pressurized tube.** (p.17) (Normal distribution). The fluctuation of pressure in a pressurized tube is due to the linear superposition of a large number of factors. The mean and standard deviation of the pressure are found to be 1.2[atm] and 0.4[atm], respectively. The tube ruptures at a pressure of 1.75[atm].

   (a) What is the probability of failure?

   (b) A pressure gauge attached to the tube reports a pressure of 0.6[atm]. Do you believe the measurement, or do you suspect that the gauge is faulty?

   (c) The tube has been changed and is now operating at a new pressure. A reliable pressure gauge is used to measure the pressure 7 times, with the results: 2.1, 2.1, 2.3, 2.4, 1.9, 2.2, 2.2 [atm]. The new tube fails at 2.6[atm]. What is the probability of failure?

7. **Adjusted machine.** (p.19) Suppose that when a machine is properly adjusted, 50% of the products are high quality and the rest are medium quality. However, when the machine is improperly adjusted, 25% of the products are high quality and the rest are medium quality. The machine is improperly adjusted 10% of the time.

   At a particular time, 5 items produced by this machine were inspected and 4 were of high quality and 1 of medium quality. What is the probability that the machine was properly adjusted when it produced these items?

8. **Passing grade.** (p.20) Consider the "American system" of examination (multiple choice) with 3 options for each of 20 questions. For each question, assume that either the student knows the answer and chooses the correct option, or he does not know the answer and chooses randomly with equal probability for each option.

   What value of a passing grade should be adopted in order that only 1% of candidates who do not know the answer to any question will pass? (Suggestion: use the normal approximation to the binomial distribution.)

9. **Switch envelopes?** (p.21) I have two envelopes, each with a positive amount of money, one with ten times more than the other. There are many such pairs of envelopes, and the probability that the pair I am holding contains \$$10^n$ and \$$10^{n+1}$ is $1/2^n$ where $n =$ 1, 2, ... This is illustrated as follows:

$$\begin{array}{rcccc} \text{Probability} & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \cdots \\ \text{Small} & \$10^1 & \$10^2 & \$10^3 & \cdots \\ \text{Large} & \$10^2 & \$10^3 & \$10^4 & \cdots \end{array} \tag{3}$$

You pick an envelope, open it, and find that it contains \$$10^n$. Do you want to switch this envelope for the other one in the pair?

10. **Gear-tooth breakage: 1.** (p.21) The stress applied to the tooth of a gear is distributed with an exponential pdf:

$$p_L(\ell) = \lambda \mathrm{e}^{-\lambda \ell}, \quad \ell \geq 0 \tag{4}$$

Likewise, the yield strength of the tooth is exponentially distributed:

$$p_y(s_y) = \mu \mathrm{e}^{-\mu s_y}, \quad s_y \geq 0 \tag{5}$$

(a) What is the probability of failure of a single specific tooth?

(b) Now consider an entire gear, with N teeth. Assume the gear fails if any single tooth fails. What is the probability of failure of the gear? What have you assumed in order to solve this?

(c) Now suppose that the gear fails only when n or more out of the N teeth fail. What is the probability of failure?

11. **Gear-tooth breakage: 2.** (p.) Consider a gear with N teeth operating at $\omega = 200$ gear cycles per minute. The load on the teeth is constant during each gear cycle, except that occasionally a single tooth gets an over-load far in excess of its yield strength, causing breakage of that tooth. The rate of occurrence of over-loads is $\lambda = 10^{-9}$ per cycle. The gear fails when 2 or more teeth are broken.

(a) What is the probability of gear failure in $T = 2000$ hours?

(b) What is the MTTF of the gear?

12. **Rockets.** (p.) A rocket is shot to a distance of about 40[km]. The payload must be activated when the rocket is 150[m] above the ground near the target. The barometric pressure is measured and used to determine the rocket altitude. However, barometric measurements are not accurate for two reasons: (1) variation of barometric pressure from day to day and (2) variation from one instrument to another as expressed by a random offset in the barometric reading of the sensor.

The first problem is solved by calibrating the barometric measurement by measuring the barometric pressure at the time of launch.

The second problem is solved by a "voting" method: N barometric sensors are used and the payload is activated when M sensors indicate that the altitude is no greater than 150[m].

Define "failure" of the activation mechanism to be: activation of the payload above 160[m] or below 140[m]. Assume that the distribution of barometric measurement error is normal with zero mean and variance $\sigma^2 = 25$[m^2].

Consider the following problems:

(a) What is the probability of failure when: $N = 3$ and $M = 1$? $N = 3$ and $M = 2$? $N = 3$ and $M = 3$?

(b) What is the utility of the marginal sensor if the payload is activated when a majority of sensors indicate an altitude no greater than 150[m]? That is, how does the probability of failure vary with N?

(c) Suppose two different types of barometric sensors are available, an inexpensive but inaccurate model costing c_1 and with variance σ_1, and an expensive but accurate model costing c_2 and with variance σ_2. Assume both types of sensors have zero-mean normal distributions. Develop an expression for choosing the equivalent number of cheap or expensive sensors, given the requirement that the probability of failure be no greater than P_{f}.

13. **Two failure mechanisms.** (p.22) A component may fail due to two different causes: excessive stress and aging. Data for the component show that the time to failure T_1 due to excessive stress is exponentially distributed:

$$f_1(t) = \lambda_1 \mathrm{e}^{-\lambda_1 t}, \quad t \geq 0 \tag{6}$$

The time to failure T_2 due to aging has a gamma distribution:

$$f_2(t) = \frac{\lambda_2}{\Gamma(k)} (\lambda_2 t)^{k-1} \mathrm{e}^{-\lambda_2 t}, \quad t \geq 0 \tag{7}$$

(a) What is the rationale for the following probability density for the time to failure T of the component? What is the meaning of the parameter p?

$$f(t) = p f_1(t) + (1-p) f_2(t), \quad t \geq 0 \tag{8}$$

(b) What is the failure rate function?

(c) What is the mean time to failure?

(d) Suppose design choices can influence the value of p. What is the optimal value of p?

14. **Crack density.** (p.24) (Poisson and exponential distributions). Solid rocket fuel has tiny internal cracks which are distributed randomly and independently in the material. Let's suppose that the average density of cracks is $\rho = 10^5$ cracks per meter^3.

    (a) What is the probability that a 1 cm^3 sample is free of cracks?

    (b) What is the probability that a 1 cm^3 sample has no more than 1 crack? Given 100 samples, each 1 cm^3, what is the probability that no sample has more than 1 crack?

    (c) What is the mean distance between cracks?

15. **Weibull distribution and the failure rate function.** (p.25) In table 1 are listed the lifetimes of 15 out of a population of 20 mechanical switches. The term ‘lifetime’ refers to the number of cycles performed before failure occurs. All other switches in the population survived more than 4050 cycles. Is the probability of failure of these switches constant, increasing or decreasing in time? Suggestion: Evaluate an empirical probability distribution, $F(t)$. Suppose the true distribution is Weibull and consider the slope of $\ln[-\ln(1-F)]$ vs $\ln t$.

Number of cycles to failure (t)	Failure number (i)
430	1
900	2
1090	3
1220	4
1500	5
1910	6
1915	7
2250	8
2600	9
2610	10
3000	11
3390	12
3430	13
3700	14
4050	15

Table 1: Failure data.