21. **Exchange rate devaluation.** (p.86) (DeGarmo, 9-30, p.400) A US firm requires a 26% rate of return in US$ on an N-year investment in a foreign country. The real return in the foreign currency in year k is $R_{r,\mathrm{for}}$. The year-0 exchange rate is $r_0 = 1$. The initial investment is S US$. There is no inflation in either country.

    (a) If the currency of the foreign country is expected to devalue at an average annual rate of 8% with respect to the US$, what rate of return in the foreign country would be required to meet the firm's requirement?

    (b) If the dollar is expected to devalue at an average annual rate of 6% with respect to the currency of the foreign country, what rate of return in the foreign country would be required to meet the firm's requirement?

Solution to Problem 21, Exchange rate devaluation (p.18). An initial US$ investment S has returns $R_{k,\mathrm{dom}}$ in US$ in years $k = 1, \ldots, N$, or returns $R_{k,\mathrm{for}}$ in the foreign currency, where:

$$R_{k,\mathrm{dom}} = r_k R_{k,\mathrm{for}} \tag{270}$$

and:

$$r_k = (1+\varepsilon)^{-k} r_0 \tag{271}$$

Recall that $r_0 = 1$ US$ per unit of foreign currency. For (a) $\varepsilon = 0.08$, and for (b) $\varepsilon = -0.06$.

The PW$_{\mathrm{dom}}$, calculated with domestic currency, is:

$$\mathrm{PW}_{\mathrm{dom}} = \sum_{k=1}^{N} (1+i_{\mathrm{dom}})^{-k} R_{k,\mathrm{dom}} \tag{272}$$

where $i_{\mathrm{dom}} = 0.26$.

The PW$_{\mathrm{for}}$, calculated with foreign currency, is:

$$\mathrm{PW}_{\mathrm{for}} = \sum_{k=1}^{N} (1+i_{\mathrm{for}})^{-k} R_{k,\mathrm{for}} \tag{273}$$

where i_{for} must be determined.

The PW's in eqs.(272) and (273) must be equal, after exchanging one of the currencies. Thus, using eqs.(270) and (271):

$$\sum_{k=1}^{N} (1+i_{\mathrm{dom}})^{-k} (1+\varepsilon)^{-k} R_{k,\mathrm{for}} = \sum_{k=1}^{N} (1+i_{\mathrm{for}})^{-k} R_{k,\mathrm{for}} \tag{274}$$

This relation holds if:

$$(1+i_{\mathrm{dom}})^{-k} (1+\varepsilon)^{-k} = (1+i_{\mathrm{for}})^{-k} \quad \text{for all } k \tag{275}$$

Thus:

$$i_{\mathrm{for}} = (1+i_{\mathrm{dom}})(1+\varepsilon) - 1 \tag{276}$$

For (a):

$$i_{\mathrm{for}} = (1+0.26)(1+0.08) - 1 = 0.3608 \tag{277}$$

For (b):

$$i_{\mathrm{for}} = (1+0.26)(1-0.06) - 1 = 0.1844 \tag{278}$$