

29. **Future foreign earnings.** (based on exam, 21.7.2014) (p.98) Your employment contract states that at the end of k years from now you will receive a payment, in \$'s, of the fixed sum A_k . The general price inflation of \$'s will be f_j in year j for $j = 1, \dots, k$. The exchange rate between \$'s and pesos at the end of year k will be r_k peso/\$. The general price inflation of pesos will be ϕ_j in year j for $j = 1, \dots, k$.

- What is the real value of A_k in \$'s at the start of year 1?
- What is the real value of A_k in pesos at the start of year 1 if you transfer the payment when it is received, which is at the end of year k ?
- The value of the payment A_k is firmly fixed by contract. However, suppose the dollar inflation rates f_j are highly uncertain, while the peso inflation rates are very stable and well known. The exchange rate r_k is also highly uncertain. Consider the following info-gap model for f_j and r_k :

$$\mathcal{U}(h) = \left\{ r_k, f_j, j = 1, \dots, k : r_k \geq 0, \left| \frac{r_k - \tilde{r}_k}{\tilde{r}_k} \right| \leq h, \left| \frac{f_j - \tilde{f}_j}{\tilde{f}_j} \right| \leq h \right\}, \quad h \geq 0 \quad (13)$$

You require that the real peso value, at the start of year 1, of the year- k earnings in \$'s, be no less than R_c . Derive an explicit algebraic expression for the robustness to uncertainty.

Solution to Problem 29, Future foreign earnings, (p.25).

(29a) The real value at time 0, in \$'s, of the \$ payment A_k at the end of year k is, from eq.(265) (see the derivation there, p.84):

$$R_{0,k} = A_{0,k} = A_k \prod_{j=1}^k (1 + f_j)^{-1} \quad (360)$$

(29b) The foreign nominal value in year k is:

$$A_{k,\text{for}} = r_k A_k \quad (361)$$

Eq.(360) applies to pesos when using the peso inflation rates, so the real value at time 0, in peso's, of the \$ payment A_k after transferring to pesos at the end of year k is:

$$R_{0,k,\text{for}} = A_{0,k,\text{for}} = A_{k,\text{for}} \prod_{j=1}^k (1 + \phi_j)^{-1} \quad (362)$$

$$= r_k A_k \prod_{j=1}^k (1 + \phi_j)^{-1} \quad (363)$$

(29c) The uncertainty in the \$ inflation is irrelevant, because the \$ inflation rate does not influence the peso value. This is because the value of A_k is fixed in dollars. However, the uncertainty in the exchange rate is relevant. The robustness is:

$$\hat{h} = \max \left\{ h : \left(\min_{r_k \in \mathcal{U}(h)} R_{0,k,\text{for}}(r_k) \right) \geq R_c \right\} \quad (364)$$

Let $m(h)$ denote the inner minimum, which occurs for the lowest exchange rate at horizon of uncertainty h : $r_k = (1 - h)^+ \tilde{r}_k$. From eq.(363):

$$m(h) = (1 - h)^+ R_{0,k,\text{for}}(\tilde{r}_k) \quad (365)$$

Equating this to R_c and solving for h yields the robustness:

$$\hat{h} = 1 - \frac{R_c}{R_{0,k,\text{for}}(\tilde{r}_k)} \quad (366)$$

or zero if this is negative.