Lecture Notes on

Price Changes: Inflation and Foreign Exchange

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Source material:

• DeGarmo, E. Paul, William G. Sullivan, James A. Bontadelli and Elin M. Wicks, 1997, *Engineering Economy.* 10th ed., chapter 9, Prentice-Hall, Upper Saddle River, NJ.

• Ben-Haim, Yakov, 2010, *Info-Gap Economics: An Operational Introduction,* Palgrave-Macmillan.

• Ben-Haim, Yakov, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty,* 2nd edition, Academic Press, London.

• Israel Central Bureau of Statistics, http://www.cbs.gov.il

A Note to the Student: These lecture notes are not a substitute for the thorough study of books. These notes are no more than an aid in following the lectures.

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[0]\lectures\Intro-Econ-DM\price-change02.tex 19.3.2025 © Yakov Ben-Haim 2024.

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1 Consumer Price Index and Inflation

§ **Consumer Price Index, CPI:**

- Measures the change over time of the price of a linearly-weighted basket of goods and services:

$$\boxed{\text{CPI} = \sum_{j=1}^{N} w_j p_j} \tag{1}$$

p_j = market price of good or service j.
w_j = weighting coefficient of good j. $w_j \geq 0$. $\sum_{j=1}^{N} w_j = 1$.

- CPI is a measure of purchasing power.

§ **Issues related to measuring the CPI:**[1]

- What goods to include? Housing? Food? Energy? Transportation? Raw materials?
- What weights to use?
- Who makes these ↑ decisions? Why does it matter?
- Is a large value of CPI desirable? For whom?
- How to measure the prices? Sample stores? Which? When? How?
- Price-sampling and data estimation may take many months or even years. CPI may be revised months or years after the fact.
- When to update the basket and its weights?
- How to compare CPIs with different baskets and weights?
- Why use linear average, rather than, for instance, geometric mean:

$$\text{CPI} = \left(\prod_{j=1}^{N} p_j \right)^{1/N} \tag{2}$$

or some other index.[2]

- Why use averages at all, rather than median or some other percentile?

[1] Central Bureau of Statistics, Statistical Abstract of Israel 2012, section 13: Prices.
[2] See: Balk, Bert, 2008, *Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference,* Cambridge University Press, Cambridge.

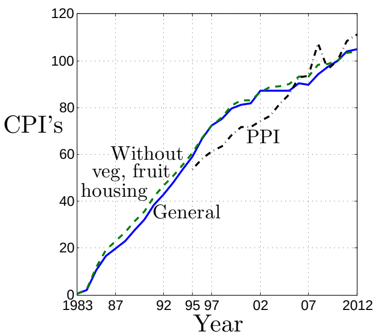

Figure 1: June CPI and PPI for Israel. Average 2010 = 100. Israel CBS. See transparency.

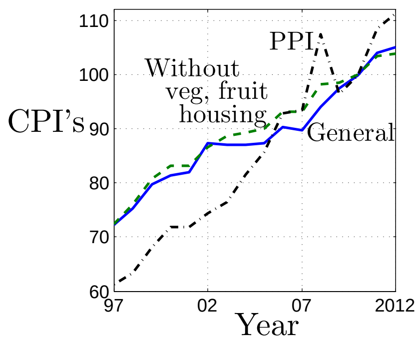

Figure 2: Expanded view of fig. 1. See transparency.

§ **Price indices compiled by the Israel Central Bureau of Statistics:**[3]

- Consumer Prices Indices:
  - **General,** figs. 1, 2, p.5, **solid blue.** Data in table 5, p.44.
  - Without vegetables and fruits.
  - Without housing
  - **Without vegetables, fruits and housing,** figs. 1, 2, **green dash.** Data in table 5, p.44.
  - Without energy.
- Prices of Dwellings
- **Producer Price Indices**—Manufacturing output for domestic market, figs. 1, 2, **dot-dash.**[4] Data in table 6, p.45.
- Index of Manufacturing Output for Exports
- Producer Price Indices for Services (SPPI)
- Prices Indices of Input in Residential Building
- Price Index of Input in Commercial and Office Building
- Price Indices of Input in Road Construction and Bridging
- Price Indices of Input in Agriculture
- Price Indices of Input in Buses
- Price Indices of Input in Public Van Services

Preliminary Questions:

- Is the difference between CPI and PPI during 1997–2002 significant?
- What's more important, the value or the slope of the price index curve?
- What caused the sharp peak in the PPI in 2008? Good news or bad?
- What is the relation between CPI and inflation?
- If there are many price indices, are there also many inflation rates?

[3] `http://www.cbs.gov.il/reader/?MIval=\%2Fprices_db\%2FPricesDB_SecondSelect_E.html&Radio1=1_3`

[4] The PPI data are for Average 2005 = 100. They have been adjusted as: PPI $\times$ $\mathrm{CPI}_{\mathrm{gen}}(2010)/\mathrm{PPI}(2005)$. Note that $\mathrm{CPI}_{\mathrm{gen}}(2010)$ is very nearly 100.

§ **Inflation for year k:**

$$\boxed{f_k = \frac{\mathrm{CPI}_k - \mathrm{CPI}_{k-1}}{\mathrm{CPI}_{k-1}}} \tag{3}$$

§ **Issues with the inflation index, f_k:**

- Since there are many CPIs, there are many inflation indices.
- There is considerable sample and estimation uncertainty in f_k, especially for the future.
- What is the best value of annual inflation? 0? +3%? −1.5%? Best for whom?

2 Nominal and Real Prices and Interest Rates

2.1 Definitions

- *Nominal dollars:*[5] The number of dollars actually involved in a transaction at the time that it occurs. Dollar bills, bank balances, and their equivalent are nominal dollars.
- *Real dollars:* Dollars expressed in the purchasing power of dollars at a reference or "base" time. Real dollars are nominal dollars after correcting for inflation.
- *General price inflation, f, f_k or $f_{k,\mathrm{gen}}$:*
  - ◦ Measure of change in purchasing power in period (e.g. year) k.
  - ◦ Based on a price index, such as eq.(3), p.6.
  - ◦ There are many price inflations, corresponding to different price indices.
- *Real interest rate, i_r:*[6] Interest rate on capital *accounting for* (removing the effect of) price inflation.
- *Nominal interest rate, i_nom:* Interest rate on capital *not accounting for* (not removing the effect of) price inflation.

[5] DeGarmo et al call this "actual dollars".
[6] DeGarmo et al call this "combined or nominal" interest rate.

2.2 Relation Between Real and Nominal Dollars

§ **Question:**

- b = index (e.g. year) of base or reference period.
- A_k = quantity of nominal dollars in period k.
- What is the real-dollar equivalent of A_k nominal dollars?

2.2.1 Purchasing Power of Real Dollars Doesn't Change Over Time Unless There is Technological Innovation and Progress

§ **Assumption:** competitive market.

§ **Example:** highly simplified.

- Consider an economy in which \$1 buys:
  a bushel of wheat, or a pound of nails, or an electric gadget.
- Suppose all prices are in equilibrium in the competitive market:
  the same effort produces the bushel of wheat, the pound of nails and the gadget.
- Now suppose innovation enables producing a pound of nails with half the effort.
- If the nail-producer doesn't cut his price in half, then
  other folks will do so and take his business away.
- If no innovations occur, then the purchasing power of \$1 is constant in time.
- Thus, for periods b through $b + j$, without innovation or progress (and no interest):

$$R_b = R_{b+1} = \cdots = R_{b+j} \tag{4}$$

  where R_k is the real dollar value of a given good or service in period k.
- We showed that an *innovation*—reducing effort—reduces the real price (of nails).
- Now consider a *regulation* or *restriction* that enlarges effort to produce nails.
  The real price of nails goes up.
- Note: regulations have goals that are often not related to the market, e.g.:
  — Health and safety of workers.
  — Environmental protection.
  — Reducing foreign competition.
  Such regulations may have adverse price-increasing effects on the market.

2.2.2 Purchasing Power (PP) of Nominal Dollars Changes Over Time Due to Inflation Even If There is No Technological Innovation or Progress

§ Re-iterate the question: what is the relation between real and nominal dollars, R_k and A_k?

§ Consider A_b nominal dollars (actual bills) in the base period b with constant inflation f.

- In eq.(3), p.6, we defined inflation as:

$$f = \frac{\mathrm{CPI}_{b+1} - \mathrm{CPI}_b}{\mathrm{CPI}_b} \tag{5}$$

- If we spend our money on the basket of goods and services, we can replace CPI by A (**Why?** See definition of CPI in eq.(1), p.4):

$$f = \frac{A_{b+1} - A_b}{A_b} \tag{6}$$

A_b = nominal \$ needed to purchase the basket in year b.

A_{b+1} = nominal \$ needed to purchase the same basket in year $b + 1$.

- Thus, from eq.(6), A_{b+1} has the same PP in period $b+1$ as A_b in period b if:

$$(1+f)^{-1}A_{b+1} = A_b \tag{7}$$

- A_{b+2} has the same PP in period $b+2$ as A_b in period b if:

$$(1+f)^{-2}A_{b+2} = A_b \tag{8}$$

- A_{b+j} has the same PP in period $b+j$ as A_b in period b if:

$$(1+f)^{-j}A_{b+j} = A_b \tag{9}$$

§ **Nominal and real dollars have the same PP in the base period:**

$$R_b = A_b \tag{10}$$

That's what we mean by the base period.

§ **Now we can answer our question,** stated at the start of section 2.2:

What is the real-dollar equivalent of A_k nominal dollars?

Combining eqs.(4) ($R_b = R_{b+1} = \cdots = R_{b+j}$), (9) and (10):

$$A_{b+j} = (1+f)^j A_b \tag{11}$$
$$= (1+f)^j R_b \tag{12}$$
$$= (1+f)^j R_{b+j} \tag{13}$$

Thus:

- If there is no technological progress, and the market is competitive, so $R_b = \cdots = R_{b+j}$.
- If inflation is positive, so $f > 0$.
- Then more dollar bills needed at $b+j$ than at b for same purchase: A_{b+j} increases with j.

Equivalently, eq.(13) implies:

$$R_{b+j} = (1+f)^{-j}A_{b+j} \tag{14}$$

Thus:

- If there is no technological progress, and the market is competitive.
- Then the real dollar value of a given nominal dollar sum decreases over time.

Question: An employer makes you a job offer, with constant monthly payments.
Do you want the contract to state the salary in real or nominal dollars?

2.2.3 Real and Nominal Income

§ DeGarmo et al, p.372.

Statement: Your starting salary is \$35,000, and will increase annually at 6% ($r = 0.06$) for 4 years. The inflation is 8% per year ($f = 0.08$). What is your nominal and your real salary each year? What is your cumulative real income?
Question: Is this a good deal for you? For the employer?
Solution:

- Your nominal salary in year j is (table 1, column 2):

$$A_j = (1+r)^{j-1} A_1 = 1.06^{j-1} \times 35,000, \quad j = 1, \ldots, 4 \tag{15}$$

- Your real salary in year j is given by eq.(14), $R_{b+j} = (1+f)^{-j} A_{b+j}$ (table 1, column 3): (Note on indices in eq.(16): $b = 1$ and $j = 1 + (j-1)$.)

$$R_j = (1+f)^{-j+1} A_j = (1+f)^{-j+1}(1+r)^{j-1} A_1 = \left(\frac{1+r}{1+f}\right)^{j-1} A_1 = \left(\frac{1.06}{1.08}\right)^{j-1} \times 35,000, \quad j = 1, \ldots, 4 \tag{16}$$

Thus real income decreases over time. (Note: $j = 1$ is the base period so $R_1 = A_1$)

- Your cumulative real income is:

$$R_{\text{tot}} = \sum_{j=1}^{4} R_j = A_1 \sum_{j=1}^{4} \left(\frac{1+r}{1+f}\right)^{j-1} = A_1 \sum_{j=0}^{3} \left(\frac{1+r}{1+f}\right)^{j} = A_1 \frac{\rho^4 - 1}{\rho - 1} \tag{17}$$

where $\rho = \frac{1+r}{1+f}$. Thus $R_{\text{tot}} = 3.8903 \times 35,000 =$ \$136,159.

Year, j	Nominal Salary, A_j	Real salary, R_j
1	35,000	35,000
2	37,100	34,352
3	39,326	33,716
4	41,686	33,091
Total		136,159

Table 1: Section 2.2.3.

§ Possible interpretations of results in table 1:

- Employer is exploiting the employee. **Why?**
- Employer anticipates that employee's effort will decrease over time.
- Given inflation f, and increasing employee productivity by 100π % each year. What would be a fair choice of r?[7]

2.2.4 Cumulative Real Income with Uncertain Inflation

§ Continue section 2.2.3.

[7] One possibility is $r = f + \pi$. Eq.(16) then implies:

$$R_j = \left(\frac{1+f+\pi}{1+f}\right)^{j-1} A_1 \tag{18}$$

Does this really achieve salary reflecting productivity? What are other models?

Statement: Your starting salary is \$35,000, and will increase annually at 6% ($r = 0.06$) for 4 years. The future inflation is estimated to be about 8% per year ($\widetilde{f} = 0.08$), plus or minus several percentage points or more. If your cumulative real income must be no less than R_c, what is your robustness to uncertain inflation? Assume that f is constant over time.

Solution:

- The info-gap model is:

$$\mathcal{U}(h) = \left\{ f : \left| \frac{f - \widetilde{f}}{s} \right| \le h \right\}, \quad h \ge 0 \tag{19}$$

with $\widetilde{f} = 0.08$ and $s = 0.25\widetilde{f}$. **Why?** Where do these numbers come from?

- The system model is from eq.(17):

$$R_{\text{tot}} = A_1 \sum_{j=0}^{3} \Bigg(\underbrace{\frac{1+r}{1+f}}_{\rho} \Bigg)^j = A_1 \frac{\rho^4 - 1}{\rho - 1} \tag{20}$$

- The performance requirement is:

$$R_{\text{tot}} \ge R_c \tag{21}$$

- The robustness is:

$$\widehat{h}(R_c) = \max \left\{ h : \left(\min_{f \in \mathcal{U}(h)} R_{\text{tot}} \right) \ge R_c \right\} \tag{22}$$

- The inner minimum, $m(h)$, occurs when $f = \widetilde{f} + sh$ (**Why?** Algebraically and intuitively):

$$m(h) = A_1 \sum_{j=0}^{3} \Bigg(\underbrace{\frac{1+r}{1+\widetilde{f}+sh}}_{\rho(h)} \Bigg)^j \tag{23}$$

$$= A_1 \frac{\rho(h)^4 - 1}{\rho(h) - 1} \tag{24}$$

Recall $m(h)$ is the inverse of $\widehat{h}(R_c)$: plot h vs $m(h)$ gives fig. 3.

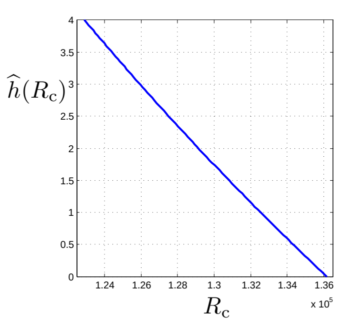

Figure 3: Robustness curve for section 2.2.4, eq.(24).

- Zeroing: no robustness at predicted cumulative real income.
- Trade off: robustness increases as critical cumulative real income decreases.
- What would be a reliable estimate of cumulative real earnings?
- The matlab code for this section is in appendix B, p.45.

2.3 Relation Between Real and Nominal Interest Rates

2.3.1 Formulation and Analysis

§ **We defined real and nominal interest rates** on p.6:

- *Real interest rate,* $i_{\rm r}$*:*[8] Interest rate on capital *accounting for* (removing the effect of) price inflation.
- *Nominal interest rate,* $i_{\rm nom}$*:* Interest rate on capital *not accounting for* (not removing the effect of) price inflation.

§ **Question:** What is the relation between $i_{\rm r}$ and $i_{\rm nom}$?

§ **Answer:**

- Consider A_{b+j} nominal dollars at period $b + j$.
- The present worth (PW) of A_{b+j} in the base period b is:

$$\boxed{{\rm PW}_b = (1 + i_{\rm nom})^{-j} A_{b+j}} \tag{25}$$

We can think of this as the **definition of the nominal interest rate.**

- Period b is the base period, so ${\rm PW}_b$ is both the nominal and the real worth.
- The real value of A_{b+j} in period $b + j$, from eq.(14), p.8, is:

$$R_{b+j} = (1 + f)^{-j} A_{b+j} \tag{26}$$

where f is the constant rate of inflation.

- The PW of R_{b+j} in the base period is:

$$\begin{aligned} {\rm PW}_b &= (1 + i_{\rm r})^{-j} R_{b+j} & (27) \\ &= (1 + i_{\rm r})^{-j} (1 + f)^{-j} A_{b+j} & (28) \end{aligned}$$

Why $i_{\rm r}$ rather than $i_{\rm nom}$ in eqs.(27) and (28)?

- Eqs.(25) and (28) are equal (**Why?**) so:

$$\begin{aligned} (1 + i_{\rm nom})^{-j} &= (1 + i_{\rm r})^{-j} (1 + f)^{-j} & (29) \\ 1 + i_{\rm nom} &= (1 + i_{\rm r})(1 + f) & (30) \\ i_{\rm nom} &= i_{\rm r} + (1 + i_{\rm r}) f & (31) \end{aligned}$$

The nominal interest rate exceeds the real interest rate if the inflation is positive.
Inverting eq.(31):

$$\boxed{i_{\rm r} = \frac{i_{\rm nom} - f}{1 + f}} \tag{32}$$

which again shows that $i_{\rm r} < i_{\rm nom}$ iff inflation if positive.

2.3.2 Real and Nominal Interest Rates

§ DeGarmo et al., pp.375–376.

§ **Statement:**

- You borrowed \$100,000 now (period $b = 0$) to be repaid in 3 years

[8] DeGarmo et al call this "combined or nominal" interest rate.

at nominal annual interest rate of 11%.

- The inflation rate is 5%.

§ **Questions:**

1. What is the nominal (actual) dollar-bill amount owed at the end of 3 years?
2. What is the real amount (value in real $) owed at the end of 3 years?
3. What is the real rate of return (real interest rate) to the lender?

§ **Answers:**

1.
   - Initial loan: A_0 =\$100,000.
   - Nominal amount due after 3 years is:

$$A_3 = (1 + i_{\text{nom}})^3 A_0 = 1.11^3 A_0 = \$136,763.10 \tag{33}$$

2. The real amount owed after 3 years is, from eq.(14), p.8:

$$R_3 = (1 + f)^{-3} A_3 = 1.05^{-3} A_3 = \$118,141.11 \tag{34}$$

3. The real rate of return (real interest rate) is, from eq.(32), p.11:

$$i_{\text{r}} = \frac{i_{\text{nom}} - f}{1 + f} = \frac{0.11 - 0.05}{1 + 0.05} = 0.05714 \tag{35}$$

Note that, as implied by eq.(32) for positive inflation f:

$$0.057 = i_{\text{r}} \quad < \quad i_{\text{nom}} = 0.11 \tag{36}$$

$$0.057 = i_{\text{r}} \quad < \quad i_{\text{nom}} - f = 0.11 - 0.05 = 0.06 \tag{37}$$

2.3.3 Investment in Equipment

§ Related to DeGarmo et al., pp.378–380.

§ **Statement:**

- New equipment will cost S = \$180,000.
- This will have revenue R = \$36,000 per year, in starting-year (real) dollars, for $N = 10$ years, with general price inflation of $f = 0.08$.
- The salvage value of the equipment at the end of 10 years will be R_s = \$30,000 in starting-year prices.
- A fixed-price maintenance contract will cost C_k = \$2,800 per year for the first 3 years and C_k = \$1,500 per year thereafter. **Question:** Are these real or nominal values?[9]
- The Minimal Acceptable Rate of Return (MARR), in nominal terms, is $i_{\text{nom}} = 0.15$.
- Ignore depreciation and taxes.

§ **Question:** Is this project economically justified?

§ **Solution:**

- The nominal cash flow in each year, from eq.(11), p.8, is (**see table 2**, col.2, p.13):

$$A_0 = -S \tag{38}$$

$$A_k = (1 + f)^k R - C_k, \quad k = 1, \ldots, 9 \tag{39}$$

$$A_{10} = (1 + f)^k (R + R_s) - C_{10} \tag{40}$$

[9] The fixed-price condition means that these are fixed dollar sums, hence nominal values.

- The real cash flow in baseline ($k = 0$) dollars, from eq.(14), p.8, is (**see table 2**, col.3, p.13):

$$R_k = (1 + f)^{-k} A_k, \quad k = 0, \ldots, 10 \tag{41}$$

- The real interest rate, after correcting the MARR for inflation, from eq.(32), p.11, is:

$$i_{\rm r} = \frac{i_{\rm nom} - f}{1 + f} = \frac{0.15 - 0.08}{1 + 0.08} = 0.0648148 \tag{42}$$

- The PW of the project, in real dollars, is:

$$\mathrm{PW} = -S + \sum_{k=1}^{N} (1 + i_{\rm r})^{-k} R_k = \$129,033.06 \tag{43}$$

This is positive so the project is economically justified.

Year, k	Nominal Revenue, A_k	Real Revenue, R_k
0	−180,000.00	−180,000
1	36,080.00	33,407.40
2	39,194.00	33,599.45
3	42,549.63	33,777.26
4	47,477.60	34,897.45
5	51,395.81	34,979.12
6	55,627.47	35,054.74
7	60,197.67	35,124.76
8	65,133.48	35,189.59
9	70,464.16	35,249.62
10	140,989.04	65,305.20

Table 2: Section 2.3.3. See transparency.

3 Engineering Decisions with Inflation

3.1 Wireless Monitoring of Distributed Servers: Multiple Inflation Indices

§ The NewTech Corporation offers a wireless monitoring system for distributed servers (e.g. residential water-use monitors, milling machines, quality control sensors, etc.).

§ **Cash flow categories:**

- S = initial capital investment.
- T_k = technical IT support in period k. Inflation rate $f_{\rm it}$: category-specific.
- P_k = replacement electronic parts in period k. Inflation rate $f_{\rm e}$: category-specific.
- M_k = maintenance cost in period k. Inflation rate $f_{\rm m}$: category-specific.
- F_k = savings in period k. Inflation rate $f_{\rm gen}$: general market rate.

§ **Inflation rates:**

- $f_{\rm gen} = 0.03$. General market rate of inflation.
- $f_{\rm it} = 0.07$. High due to rapid growth in the hitech sector.
- $f_{\rm e} = 0.05$. Moderate, above general rate.
- $f_{\rm m} = 0.03$. Equal to general inflation rate of market.

§ **Interest rates:**

- $i_{\rm nom}$ = nominal interest rate, not correcting for inflation. Defined on p.6 and in eq.(25), p.11.
- MARR = $i_{\rm nom}$ = 0.15.
- $i_{\rm r}$ = real interest rate, correcting for inflation. Defined on p.6 and eq.(35), p.12.

§ **Question:**

- Assuming that:
  - T_k, P_k, M_k and F_k are constant in real dollars, each wrt its own inflation rate, and:
    $S = \$100{,}000$.
    $T_0 = \$2{,}000$.
    $P_0 = \$5{,}000$.
    $M_0 = \$3{,}000$.
    $F_0 = \$30{,}000$.
    The subscript 0 in these 4 items is for notational convenience later.
  - The real value of the net present worth each year is calculated with the general inflation rate, f_{gen}.
- Is this project economically justified if its life is $N = 10$ years?

§ **Solution:** Calculate PW, accounting for the category-specific inflation rates.

- A_k = nominal cash flow in period $k = 0, \ldots, N$:

$$A_0 = -S \tag{44}$$

$$A_k = (1+f_{\text{gen}})^k F_0 - (1+f_{\text{it}})^k T_0 - (1+f_{\text{e}})^k P_0 - (1+f_{\text{m}})^k M_0, \quad k = 1, \ldots, N \tag{45}$$

The idea in eq.(45) is that:
  - Expenses T_k, P_k and M_k each increases over time at its own inflation rate.
  - Savings F_k are cash balances that are used for general purposes so their value inflates at the general market rate.
- R_k = real value of net worth in year k, relative to the general market value of a dollar, is:

$$R_k = (1+f_{\text{gen}})^{-k} A_k \tag{46}$$

- The real interest rate, eq.(35), p.12, is:

$$i_{\text{r}} = \frac{i_{\text{nom}} - f_{\text{gen}}}{1+f_{\text{gen}}} \tag{47}$$

- The real present worth, wrt the firm's MARR ($= i_{\text{nom}}$), is:

$$\text{PW} = -S + \sum_{k=1}^{N} (1+i_{\text{r}})^{-k} R_k \tag{48}$$

$$= -S + \sum_{k=1}^{N} (1+i_{\text{r}})^{-k} (1+f_{\text{gen}})^{-k} A_k \tag{49}$$

$$= -S + \sum_{k=1}^{N} (1+i_{\text{r}})^{-k} (1+f_{\text{gen}})^{-k} \Big[(1+f_{\text{gen}})^k F_0 - (1+f_{\text{it}})^k T_0 - (1+f_{\text{e}})^k P_0 - (1+f_{\text{m}})^k M_0 \Big] \tag{50}$$

$$= -S + \sum_{k=1}^{N} (1+i_{\text{r}})^{-k} \left[F_0 - \left(\frac{1+f_{\text{it}}}{1+f_{\text{gen}}} \right)^k T_0 - \left(\frac{1+f_{\text{e}}}{1+f_{\text{gen}}} \right)^k P_0 - M_0 \right] \tag{51}$$

$$= -S + \sum_{k=1}^{N} (1+i_{\text{r}})^{-k} \Big[F_0 - \rho_{\text{it}}^k T_0 - \rho_{\text{e}}^k P_0 - M_0 \Big] \tag{52}$$

which defines ρ_{it} and ρ_{e}. Recall $f_{\text{m}} = f_{\text{gen}}$.

§ **Results:**

- The 4 terms in square brackets in eq.(52), showing **nominal inflation,** are shown in table 3.

Year, k	F_0	$\rho_{\rm it}^k T_0$	$\rho_{\rm e}^k P_0$	M_0
1	30,000	2,078	5,097	3,000
2	30,000	2,158	5,196	3,000
3	30,000	2,242	5,297	3,000
4	30,000	2,329	5,400	3,000
5	30,000	2,420	5,505	3,000
6	30,000	2,514	5,612	3,000
7	30,000	2,611	5,721	3,000
8	30,000	2,713	5,832	3,000
9	30,000	2,818	5,945	3,000
10	30,000	2,927	6,060	3,000

Table 3: Section 3.1. See transparency.

- The 4 terms in the summation in eq.(52), showing **real PW,** are shown in table 4.

Year, k	$(1+i_{\rm r})^{-k} F_0$	$(1+i_{\rm r})^{-k}\rho_{\rm it}^k T_0$	$(1+i_{\rm r})^{-k}\rho_{\rm e}^k P_0$	$(1+i_{\rm r})^{-k} M_0$
1	26,870	1,861	4,565	2,687
2	24,066	1,731	4,168	2,407
3	21,555	1,611	3,806	2,155
4	19,305	1,499	3,475	1,931
5	17,291	1,395	3,173	1,729
6	15,487	1,298	2,897	1,549
7	13,871	1,207	2,645	1,387
8	12,423	1,123	2,415	1,242
9	11,127	1,045	2,205	1,113
10	9,966	972	2,013	997
Total	\$171,959.64	\$13,742.82	\$31,361.52	\$17,195.96

Table 4: Section 3.1. See transparency.

- The real PW, eq.(52), is:

$$\text{PW} = -100,000 + 171,959.64 - 13,742.82 - 31,361.52 - 17,195.96 = \$9,659.34 \tag{53}$$

The PW is positive so the project is economically justified.

3.2 Wireless Monitoring of Distributed Servers: Multiple Inflation Indices, Continued

- Continue section 3.1 and **explore the question:**
  Is the project economically justified for durations N from 5 to 20 years?
- The PW, eq.(52), p.14, vs. N, for the same parameters as section 3.1, in fig. 4, p.16.[10]
  - PW < 0 for $N \leq 8$ years. Not economically justified.
  - PW > 0 for $N \geq 9$ years. Economically justified.
  - Increasing PW as N increases.
  - Diminishing marginal PW as N increases.

    Question: What does this mean? What causes it? (See eq.(52), p.14.)

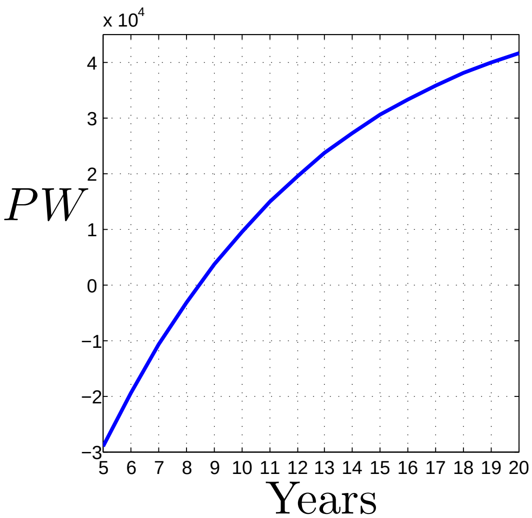

Figure 4: PW vs N for section 3.1, eq.(52).

[10]Data in table 7, p.46.

3.3 Diminishing Marginal Utility and the Petersburg Paradox

§ Fig. 4 motivates a brief discussion of diminishing marginal utility.

3.3.1 Petersburg Paradox

§ We begin by introducing the **St Petersburg Paradox,** discussed by Daniel Bernoulli in 1738.

- **Worthwhile bet:**
  If the average return from a bet exceeds the cost of betting, then the bet is worthwhile.
- **The game:**
  - Use a fair coin: $p_{\text{heads}} = p_{\text{tails}} = 0.5$.
  - The pot starts at \$1 and is doubled each time 'heads' appears.
  - The game ends on first appearance of 'tails', and the player takes the pot.
  - If the game ends on step n, then the gain is 2^{n-1}.
  - The probability that the game ends on step n is $\left(\frac{1}{2}\right)^n$.
  - The expected return is:

$$\text{E} = \sum_{n=1}^{\infty} 2^{n-1} \left(\frac{1}{2}\right)^n = \frac{1}{2} \sum_{n=1}^{\infty} 1 = \infty \tag{54}$$

- **The paradox:** This is a worthwhile bet at any cost.
  Folks should be willing to pay any amount in order to play the game! **Would you?**
- **Bernoulli's solution:**
  - The utility, $u(x)$, of winning increases by a diminishing increment
    as the size of the pot increases.
    Why? Does decreasing marginal utility make sense?
  - Thus the expected utility is (or can be) finite:

$$\text{EU} = \sum_{n=1}^{\infty} u(2^n) \left(\frac{1}{2}\right)^n < \infty \tag{55}$$

○ **Question:** Are there utility functions with decreasing marginal utility for which EU in eq.(55) is infinite?

§ **Fig. 4, p.16 illustrates:** The present worth increases with diminishing marginal utility.

3.3.2 Petersburg Paradox: Info-Gap Robustness

§ **Uncertain probability of tails:**

$\widetilde{p}$ = our best guess of the probability of tails on each coin flip. E.g. $\widetilde{p} = \frac{1}{2}$.
p = unknown true probability of tails. Could be better (bigger) or worse (smaller).
w = error estimate (not worst case) of $\widetilde{p}$.

§ **Info-gap model** for uncertain probability of tails:

$$\mathcal{U}(h) = \left\{ p : \ p \in [0, \ 1], \ \left| \frac{p - \widetilde{p}}{w} \right| \le h \right\}, \quad h \ge 0 \tag{56}$$

§ **Expected utility** when terminating on first occurrence of tails:

$$\mathrm{EU}(p) = \sum_{n=1}^{\infty} 2^{n-1} p^n \tag{57}$$

§ **Performance requirement:**

$\mathrm{EU}(\widetilde{p})$ would be OK (e.g. ∞), but we'd accept a lower value, $\mathrm{EU_c}$:

$$\mathrm{EU}(p) \ge \mathrm{EU_c} \tag{58}$$

§ **Robustness:**

$$\widehat{h}(\mathrm{EU_c}) = \max \left\{ h : \ \left(\min_{p \in \mathcal{U}(h)} \mathrm{EU}(p) \right) \ge \mathrm{EU_c} \right\} \tag{59}$$

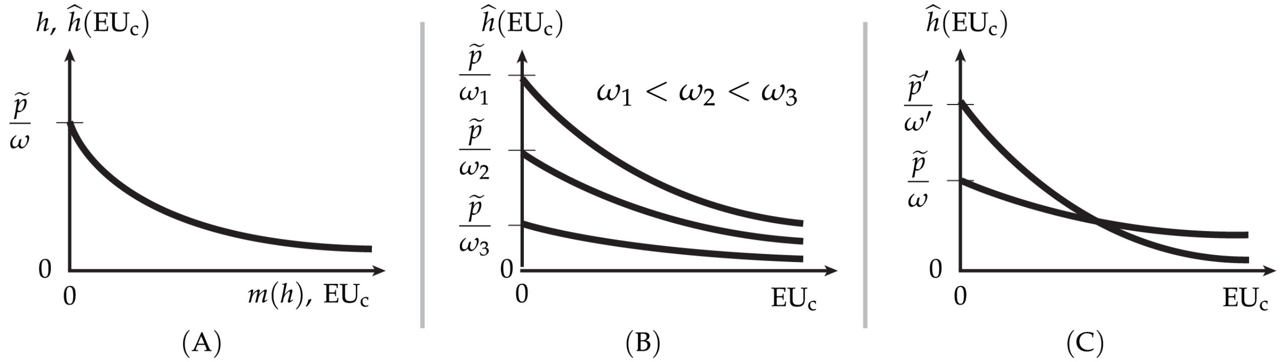

Figure 5: Schematic illustration of robustness curves for the Petersburg paradox.

§ **Evaluating the robustness:**

•Define $m(h)$ as inner minimum in definition of $\widehat{h}(\mathrm{EU_c})$:

$$\widehat{h}(\mathrm{EU_c}) = \max\left\{h:\ \left(\min_{p\in\mathcal{U}(h)} \mathrm{EU}(p)\right) \ge \mathrm{EU_c}\right\} \tag{60}$$

- $m(h) = \min\limits_{p\in\mathcal{U}(h)} \sum\limits_{n=1}^{\infty} 2^{n-1}p^n$.
- $m(h)$ is inverse function of $\widehat{h}(\mathrm{EU_c})$.
- Minimum occurs for $p = (\widetilde{p} - wh)^+$. (Exponent '+': constrained to [0,1].) Thus, for $h \le \widetilde{p}/w$:

$$m(h) = \sum_{n=1}^{\infty} 2^{n-1}(\widetilde{p} - wh)^n \tag{61}$$

- Schematic robustness curve in fig. 5A, p.19.
- An info-gap resolution of the Petersburg paradox:
  - ◦ A bet is justified if it has large robustness, no less than $\widehat{h}_c$.
  - ◦ This robustness is obtained only for EU up to a finite value, $\mathrm{EU_c}$.
  - ◦ Thus only bets up to this finite magnitude, $\mathrm{EU_c}$, are justified.
- Robustness decreases as uncertainty weight, w, increases. Fig. 5B, p.19. **Why?**
- Note: $m(0) = \sum\limits_{n=1}^{\infty} 2^{n-1}\widetilde{p}$ which may be infinite, e.g. when $\widetilde{p} = \frac{1}{2}$.

  In this case, $\widehat{h}(\mathrm{EU_c}) \to 0$ as $\mathrm{EU_c} \to \infty$: positive robustness at all finite critical utilities.

- Is there the possibility of a decision dilemma when choosing between $(\widetilde{p}, w)$ and $(\widetilde{p}', w')$?

  "Prime" putatively better but more uncertain.

  See fig. 5C, p.19.
- If so, then the resolution: Preference reversal.

3.3.3 Petersburg Paradox: Info-Gap Opportuneness

§ **Uncertainty** can be **pernicious** (motivating **robustness**), or
propitious (motivating **opportuneness**).

§ **Same info-gap model** for both situations. Non-probabilistic, unbounded uncertainty:

$$\mathcal{U}(h) = \left\{ p : \ p \in [0,\ 1], \ \left| \frac{p - \widetilde{p}}{w} \right| \le h \right\}, \quad h \ge 0 \tag{62}$$

Question: What does "unbounded" mean, when p is constrained to $[0, 1]$?[11]

§ Analogous **performance requirements** with different meanings:
Robustness *requirement*: $\mathrm{EU} \ge \mathrm{EU_c}$
Opportuneness *aspiration*: $\mathrm{EU} \ge \mathrm{EU_w}$ where $\mathrm{EU_w} > \mathrm{EU_c}$.

§ **Robustness:** *maximum* horizon of uncertainty at which worst outcome is *acceptable:*

$$\widehat{h}(\mathrm{EU_c}) = \max \left\{ h : \ \left(\min_{p \in \mathcal{U}(h)} \mathrm{EU}(p) \right) \ge \mathrm{EU_c} \right\} \tag{63}$$

Maximum horizon of uncertainty at which *requirement* is *guaranteed.*

§ **Opportuneness:** *minimum* horizon of uncertainty at which best outcome is *wonderful:*

$$\widehat{\beta}(\mathrm{EU_w}) = \min \left\{ h : \ \left(\max_{p \in \mathcal{U}(h)} \mathrm{EU}(p) \right) \ge \mathrm{EU_w} \right\} \tag{64}$$

Minimum horizon of uncertainty at which *aspiration* is *possible.*

§ **Complementary** immunity functions.

[11] Unbounded in the domain of probabilities.

§ **Evaluating the opportuneness:**

•Define $M(h)$ as inner maximum in definition of $\widehat{\beta}(\mathrm{EU_w})$:

$$\widehat{\beta}(\mathrm{EU_w}) = \min \left\{ h : \left(\max_{p \in \mathcal{U}(h)} \mathrm{EU}(p) \right) \geq \mathrm{EU_w} \right\} \tag{65}$$

- $M(h) = \max_{p \in \mathcal{U}(h)} \sum_{n=1}^{\infty} 2^{n-1} p^n$.
- $M(h)$ is inverse function of $\widehat{\beta}(\mathrm{EU_w})$.
- Maximum occurs for $p = (\widetilde{p} + wh)^+$. Thus, for $h \leq \frac{1-\widetilde{p}}{w}$:

$$M(h) = \sum_{n=1}^{\infty} 2^{n-1} (\widetilde{p} + wh)^n \tag{66}$$

- Estimated EU is $\widetilde{\mathrm{EU}} = M(0)$. May be infinite, e.g. if $\widetilde{p} = 0.5$.
  In this case, $\widehat{\beta}(\mathrm{EU_w}) = 0$ for all $\mathrm{EU_w}$: uncertainty not needed to enable wonderful windfall.
- Schematic opportuneness curve in fig. 6, p.21, if $M(0) < \infty$.

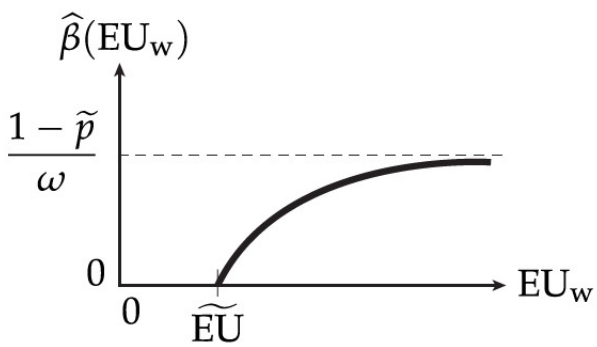

Figure 6: Schematic illustration of opportuneness curve for the Petersburg paradox.

§ **Robustness or opportuneness?**

- Risk-averse decision maker may prefer robustness against pernicious uncertainty.
- Risk-loving decision maker may prefer opportuneness from propitious uncertainty.
- Which are you? Does it depend on the decision?

3.4 Choosing Between Two Technologies

§ **Background:**

- The BrandTech Corporation offers a wireless monitoring system using different technology from NewTech's model (sections 3.1 and 3.2).
- BrandTech's model:
  - Costs more up front.
  - Uses less hitech and spare parts, with high inflation rates.
  - Uses more low-tech maintenance, with low inflation rates.
- You must choose between these design concepts.

§ **Cash flow categories:**

- S = initial capital investment.
- T_k = technical IT support in period k. Inflation rate $f_{\rm it}$: category-specific.
- P_k = replacement electronic parts in period k. Inflation rate $f_{\rm e}$: category-specific.
- M_k = maintenance cost in period k. Inflation rate $f_{\rm m}$: category-specific.
- F_k = savings in period k. Inflation rate $f_{\rm gen}$: general market rate.

§ **Inflation rates:**

- $f_{\rm gen} = 0.03$. General market rate of inflation.
- $f_{\rm it} = 0.07$. High due to rapid growth in the hitech sector.
- $f_{\rm e} = 0.05$. Moderate, above general rate.
- $f_{\rm m} = 0.03$. Equal to general inflation rate of market.

§ **Interest rates:**

- $i_{\rm nom}$ = nominal interest rate, not correcting for inflation. Defined on p.6 and in eq.(25), p.11.
- MARR = $i_{\rm nom} = 0.15$.
- $i_{\rm r}$ = real interest rate, correcting for inflation. Defined on p.6 and eq.(35), p.12.

§ **Question:**

- Assuming for BrandTech that:
  - T_k, P_k, M_k and F_k are constant in real dollars, each wrt its own inflation rate, and:
    S = \$110,000. NewTech: \$100,000.
    T_0 = \$1,400. NewTech: \$2,000.
    P_0 = \$3,500. NewTech: \$5,000.
    M_0 = \$3,500. NewTech: \$3,000.
    F_0 = \$30,000. NewTech: \$30,000.
    The subscript 0 in these 4 items is for notational convenience later.
  - The real value of the net present worth each year is calculated with the general inflation rate, $f_{\rm gen}$.
- Which technology is economically preferable between for $N = 5, \ldots, 20$ years?

§ **Solution:**

- Use PW(N) for NewTech from section 3.2, p.16. Fig. 4, p.16 and table 7, p.46.
- Calculate PW(N) for BrandTech, accounting for the category-specific inflation rates. Use eqs.(51) and (52), p.14:

$$\text{PW} = -S + \sum_{k=1}^{N} (1+i_{\rm r})^{-k} \left[F_0 - \left(\frac{1+f_{\rm it}}{1+f_{\rm gen}} \right)^k T_0 - \left(\frac{1+f_{\rm e}}{1+f_{\rm gen}} \right)^k P_0 - M_0 \right] \tag{67}$$

$$= -S + \sum_{k=1}^{N} (1+i_{\rm r})^{-k} \left[F_0 - \rho_{\rm it}^k T_0 - \rho_{\rm e}^k P_0 - M_0 \right] \tag{68}$$

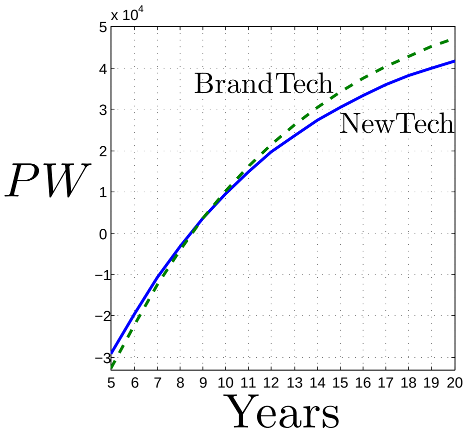

Figure 7: PW vs N for section 3.4, eq.(52).

§ **Results,** fig. 7, p.23. NewTech data in table 7, p.46. BrandTech data in table 8, p.46:

- Neither technology is economically sustainable for $N < 9$ years:

$$0 > \mathrm{PW}_{\mathrm{NewTech}}(N) > \mathrm{PW}_{\mathrm{BrandTech}}(N), \quad N < 9 \text{ years} \tag{69}$$

- Both technologies are economically sustainable for $N \geq 9$ years:

$$0 < \mathrm{PW}_{\mathrm{NewTech}}(N) \quad < \quad \mathrm{PW}_{\mathrm{BrandTech}}(N), \quad N > 9 \text{ years} \tag{70}$$

- The two technologies are essentially economically equivalent at $N = 9$:

$$\mathrm{PW}_{\mathrm{BrandTech}}(9) = \$3611, \quad \mathrm{PW}_{\mathrm{NewTech}}(9) = \$3676 \tag{71}$$

- The relative advantage of BrandTech increases with life, reaching maximum at $N = 20$:

$$\frac{\mathrm{PW}_{\mathrm{BrandTech}}(20) - \mathrm{PW}_{\mathrm{NewTech}}(20)}{\mathrm{PW}_{\mathrm{BrandTech}}(20)} = \frac{\$5,506}{\$47,267} = 0.116 \tag{72}$$

- Explanation:
  - ◦ Recall eq.(67), p.22: $\mathrm{PW} = -S + \sum_{k=1}^{N}(1+i_{\mathrm{r}})^{-k}\left[F_0 - \left(\frac{1+f_{\mathrm{it}}}{1+f_{\mathrm{gen}}}\right)^k T_0 - \left(\frac{1+f_{\mathrm{e}}}{1+f_{\mathrm{gen}}}\right)^k P_0 - M_0\right]$.
  - ◦ BrandTech is preferred for large N because
    its price structure is biased to lower inflation,
    even though its initial capital cost is greater.
  - ◦ NewTech is preferred for small N because
    its price structure is biased to higher inflation,
    and its initial capital cost is lower.

3.5 Wireless Monitoring of Distributed Servers: Multiple Uncertain Inflation Indices

§ Return to section 3.1, p.13, and consider uncertain inflation rates.

§ **Cash flow categories:**

- S = initial capital investment.
- T_k = technical IT support in period k. Inflation rate f_{it}: category-specific.
- P_k = replacement electronic parts in period k. Inflation rate f_{e}: category-specific.
- M_k = maintenance cost in period k. Inflation rate f_{m}: category-specific.
- F_k = savings in period k. Inflation rate f_{gen}: general market rate.

§ **Interest rates:**

- i_{nom} = nominal interest rate, not correcting for inflation. Defined on p.6 and in eq.(25), p.11.
- Choose i_{nom} as the MARR = 0.15, so $i_{\text{nom}} = 0.15$.
- i_{r} = real interest rate, correcting for inflation. Defined on p.6 and eq.(35), p.12.

§ **Estimated inflation rates:**

- $\widetilde{f}_{\text{gen}} = 0.03$. General market rate of inflation.
- $\widetilde{f}_{\text{it}} = 0.07$. High due to rapid growth in the hitech sector.
- $\widetilde{f}_{\text{e}} = 0.05$. Moderate, above general rate.
- $\widetilde{f}_{\text{m}} = 0.03$. Equal to general inflation rate of market.

§ **Contextual information** and understanding about future sectoral inflation:

- All inflation rates are **constant over time.**
- The constant values of these inflation rates are **uncertain**.
- f_{gen} and f_{m} will continue to be linked: $f_{\text{m}} = f_{\text{gen}}$.
  - The estimated value, $\widetilde{f}_{\text{gen}}$, may err by about half a percentage point or maybe more.
  - The estimated values may be either under- or over-estimates.
- $\widetilde{f}_{\text{it}}$ may be an under-estimate of f_{it}, but it is not an over-estimate. That is, $f_{\text{it}} \geq \widetilde{f}_{\text{it}}$.
  $\widetilde{f}_{\text{it}}$ may under-estimate f_{it} by several percentage points or more.
- $\widetilde{f}_{\text{e}}$ may be an under-estimate of f_{e}, but it is not an over-estimate. That is, $f_{\text{e}} \geq \widetilde{f}_{\text{e}}$.
  $\widetilde{f}_{\text{e}}$ may under-estimate f_{e} by 1 or 2 percentage points or more.

§ **Error estimates of inflation rates:**

- $s_{\text{gen}} = 0.005$. Symmetric uncertainty.
- $s_{\text{it}} = 0.03$. Asymmetric uncertainty.
- $s_{\text{e}} = 0.02$. Asymmetric uncertainty.

§ **Questions:**

- Assuming that:
  - T_k, P_k, M_k and F_k are constant in real dollars, each wrt its own inflation rate, and:
    S = \$100,000.
    T_0 = \$2,000.
    P_0 = \$5,000.
    M_0 = \$3,000.
    F_0 = \$30,000.
    The subscript 0 in these 4 items is for notational convenience later.
  - The real value of the net present worth each year is calculated with
    the general inflation rate, f_{gen}.
- Is this project economically justified if its life is $N = 10$ years?
- For what lifetimes, N, would this project be economically justified?
- For what initial capital investment, S, would this project be economically justified?

§ **Solution:** Evaluate robustness of PW to uncertainty in the category-specific inflation rates.

- **The present worth,** from eq.(51), p.14:

$$\text{PW} \;=\; -S + \sum_{k=1}^{N} (1+i_{\text{r}})^{-k} \left[F_0 - \left(\frac{1+f_{\text{it}}}{1+f_{\text{gen}}} \right)^k T_0 - \left(\frac{1+f_{\text{e}}}{1+f_{\text{gen}}} \right)^k P_0 - M_0 \right] \tag{73}$$

- **The real interest rate,** eq.(35), p.12, is:

$$i_{\text{r}} = \frac{i_{\text{nom}} - f_{\text{gen}}}{1+f_{\text{gen}}} \tag{74}$$

Thus:

$$1+i_{\text{r}} = 1 + \frac{i_{\text{nom}} - f_{\text{gen}}}{1+f_{\text{gen}}} = \frac{1+i_{\text{nom}}}{1+f_{\text{gen}}} \tag{75}$$

Hence:

$$(1+i_{\text{r}})^{-k} \left(\frac{1+f_{\text{it}}}{1+f_{\text{gen}}} \right)^k \;=\; \left(\frac{1+f_{\text{gen}}}{1+i_{\text{nom}}} \frac{1+f_{\text{it}}}{1+f_{\text{gen}}} \right)^k = \left(\frac{1+f_{\text{it}}}{1+i_{\text{nom}}} \right)^k \tag{76}$$

$$(1+i_{\text{r}})^{-k} \left(\frac{1+f_{\text{e}}}{1+f_{\text{gen}}} \right)^k \;=\; \left(\frac{1+f_{\text{gen}}}{1+i_{\text{nom}}} \frac{1+f_{\text{e}}}{1+f_{\text{gen}}} \right)^k = \left(\frac{1+f_{\text{e}}}{1+i_{\text{nom}}} \right)^k \tag{77}$$

- **Now the PW** in eq.(73) is:

$$\text{PW} = -S + \sum_{k=1}^{N} \left[\left(\frac{1+f_{\text{gen}}}{1+i_{\text{nom}}} \right)^k (F_0 - M_0) - \left(\frac{1+f_{\text{it}}}{1+i_{\text{nom}}} \right)^k T_0 - \left(\frac{1+f_{\text{e}}}{1+i_{\text{nom}}} \right)^k P_0 \right] \tag{78}$$

- **Info-gap model** for uncertain inflation rates:

$$\mathcal{U}(h) = \left\{ f = (f_{\text{it}}, f_{\text{e}}, f_{\text{gen}}) : \; 0 \le \frac{f_{\text{it}} - \widetilde{f}_{\text{it}}}{s_{\text{it}}} \le h, \; 0 \le \frac{f_{\text{e}} - \widetilde{f}_{\text{e}}}{s_{\text{e}}} \le h, \; \left| \frac{f_{\text{gen}} - \widetilde{f}_{\text{gen}}}{s_{\text{gen}}} \right| \le h \right\}, \quad h \ge 0 \tag{79}$$

- **Performance requirement:**

$$\text{PW}(f) \ge PW_{\text{c}} \tag{80}$$

- **Robustness:**

$$\widehat{h}(PW_{\text{c}}) = \max \left\{ h : \; \left(\min_{f \in \mathcal{U}(h)} \text{PW}(f) \right) \ge PW_{\text{c}} \right\} \tag{81}$$

- **Inner minimum,** $m(h)$**, occurs at** (assuming $T_0 \ge 0$ and $P_0 \ge 0$):

$$f_{\text{it}}(h) \;=\; \widetilde{f}_{\text{it}} + s_{\text{it}} h \tag{82}$$

$$f_{\text{e}}(h) \;=\; \widetilde{f}_{\text{e}} + s_{\text{e}} h \tag{83}$$

  - If $F_0 \ge M_0$ then inner minimum at **minimal** f_{gen}:
    Note: f cannot be ≤ -1 or $R \to \infty$, see eq.(14), p.8. Thus:

$$f_{\text{gen}}(h) = \begin{cases} \widetilde{f}_{\text{gen}} - s_{\text{gen}} h, & \text{if } h < \dfrac{1+\widetilde{f}_{\text{gen}}}{s_{\text{gen}}} \\ 0, & \text{else} \end{cases} \tag{84}$$

  - If $F_0 < M_0$ then inner minimum at **maximal** f_{gen}:

$$f_{\text{gen}}(h) = \widetilde{f}_{\text{gen}} + s_{\text{gen}} h \tag{85}$$

- **Inner minimum equals,** using eqs.(82)–(85) in eq.(78):

$$m(h) = -S + \sum_{k=1}^{N} \left[\left(\frac{1+f_{\text{gen}}(h)}{1+i_{\text{nom}}} \right)^k (F_0 - M_0) - \left(\frac{1+f_{\text{it}}(h)}{1+i_{\text{nom}}} \right)^k T_0 - \left(\frac{1+f_{\text{e}}(h)}{1+i_{\text{nom}}} \right)^k P_0 \right] \tag{86}$$

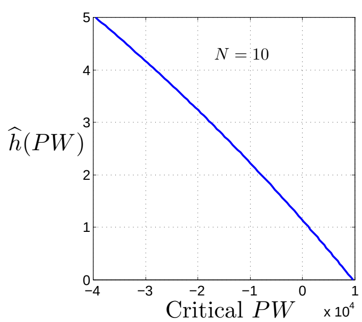

Figure 8: Robustness curve for $N = 10$ years, section 3.5, eq.(86). $S =$ \$100,000. See transp.

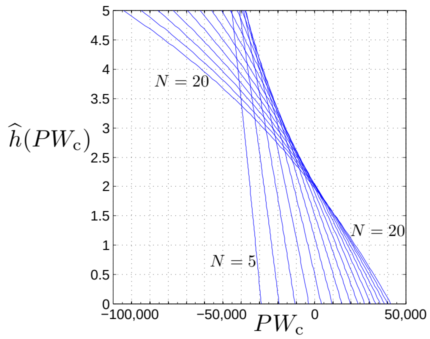

Figure 9: Robustness curves for $N = 5$ to 20 years, section 3.5, eq.(86). $S =$ \$100,000. See transp.

§ **Results:**[12]

- Is this project economically justified if its life is $N = 10$ years? Robustness curve in fig. 8.
  - **Zeroing:** $\widehat{h}(\text{PW} = \$9,659) = 0$, as in eq.(53), p.15.
  - **Trade off:**
    - — $\widehat{h}(PW_c = 0) = 1.1$. Marginally acceptable robustness at break-even PW.
    - — $\widehat{h}(PW_c) = 3$ at $PW_c = -\$17{,}600$. Large robustness at large loss.
  - This lifetime does not look economically feasible after considering uncertainty:
    Cost of robustness is too high. **Question:** What does "cost of robustness" mean?
- For what lifetimes, N, would this project be economically justified?
  Robustness curves in fig. 9, p.26:
  - **Zeroing:** Horizontal intercepts ($\widehat{h} = 0$) at nominal PW's of table 7, p.46.
    - — Nominal PW > 0 for $N \geq 9$ years, as in section 3.2, p.16.
    - — Nominal feasibility for $N \geq 9$ years.
  - **Trade off:**

$$\widehat{h}(PW_c) = 1 \implies PW_c \geq 0 \text{ for } N \geq 10 \tag{87}$$

$$\widehat{h}(PW_c) = 2 \implies PW_c \approx 0 \text{ for } N \geq 14 \tag{88}$$

$$\widehat{h}(PW_c) = 3 \implies PW_c \geq 0 \text{ for no } N \leq 20 \tag{89}$$

  Marginally feasible only for $N \geq 14$.

  - **Cost of robustness:**

$$\text{Low cost: } \frac{\Delta\widehat{h}}{\$1,000} \approx 0.3 \quad \text{for } N = 5 \tag{90}$$

$$\text{Med cost: } \frac{\Delta\widehat{h}}{\$1,000} \approx 0.1 \quad \text{for } N = 10 \tag{91}$$

$$\text{High cost: } \frac{\Delta\widehat{h}}{\$1,000} \approx 0.03 \quad \text{for } N = 20 \tag{92}$$

  Economic explanation:
  - — Future inflation is highly uncertain.
  - — Long and far future is more vulnerable than short and near future.
  - — Hence cost of robustness increases with N.

[12] Calculations with GapZapper application Econ-Dec-Making-Course: Price-Change01

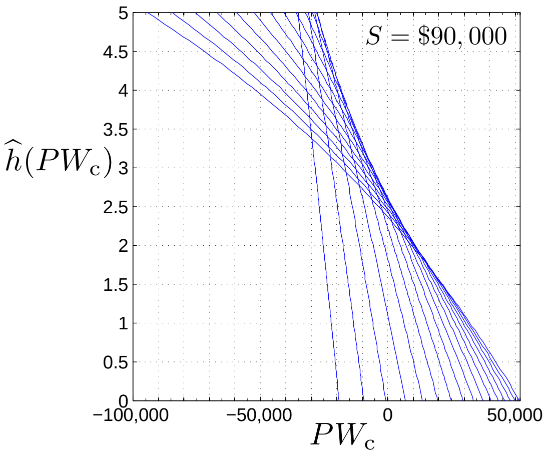

Figure 10: Robustness curves for $N = 5$ to 20 years, section 3.5, eq.(86). $S = \$90{,}000$. See transp.

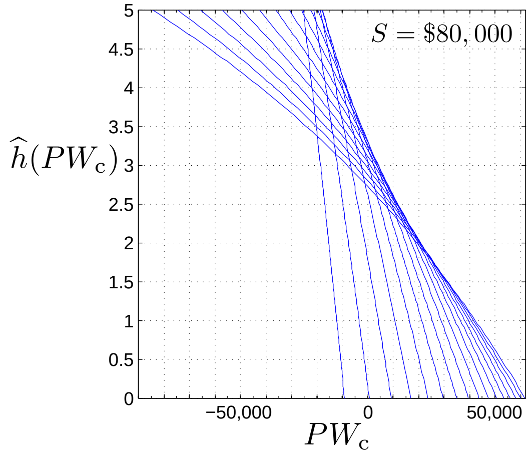

Figure 11: Robustness curves for $N = 5$ to 20 years, section 3.5, eq.(86). $S = \$80{,}000$. See transp.

- For what initial capital investment, S, would this project be economically justified?
  - From eq.(86), p.25: capital investment increase ΔS shifts robustness curve left by ΔS.
  - Compare figs. 10 and 11 against 9.

    For $S = \$100{,}000$, eqs.(87)–(89), p.26:

$$\widehat{h}(PW_c) = 1 \implies PW_c \geq 0 \text{ for } N \geq 10 \tag{93}$$

$$\widehat{h}(PW_c) = 2 \implies PW_c \approx 0 \text{ for } N \geq 14 \tag{94}$$

$$\widehat{h}(PW_c) = 3 \implies PW_c \geq 0 \text{ for no } N \leq 20 \tag{95}$$

For $S = \$90{,}000$:

$$\widehat{h}(PW_c) = 1 \implies PW_c \geq 0 \text{ for } N \geq 8 \tag{96}$$

$$\widehat{h}(PW_c) = 2 \implies PW_c \geq 0 \text{ for } N \geq 10 \tag{97}$$

$$\widehat{h}(PW_c) = 3 \implies PW_c \geq 0 \text{ for no } N \leq 20 \tag{98}$$

For $S = \$80{,}000$:

$$\widehat{h}(PW_c) = 1 \implies PW_c \geq 0 \text{ for } N \geq 7 \tag{99}$$

$$\widehat{h}(PW_c) = 2 \implies PW_c \geq 0 \text{ for } N \geq 8 \tag{100}$$

$$\widehat{h}(PW_c) = 3 \implies PW_c \approx 0 \text{ for } 8 \leq N \leq 16 \tag{101}$$

If we require robustness of 2 (moderate immunity to uncertainty), then:

- Require $N > 14$ for $S = \$100,000$, eq.(94).
- Require $N > 10$ for $S = \$90,000$, eq.(97).
- Require $N > 8$ for $S = \$80,000$, eq.(100).

3.6 Choosing Between Two Technologies with Multiple Uncertain Inflation Indices

§ **Background.** Return to section 3.4, p.22:

- The BrandTech Corporation offers a wireless monitoring system using different technology from NewTech's model (sections 3.1 and 3.2, pp.13, 16).
- BrandTech's model:
  - Costs more up front.
  - Uses less hitech and spare parts, with high inflation rates.
  - Uses more low-tech maintenance, with low inflation rates.
- You must:
  - Choose between these design concepts.
  - Specify a feasible lifetime.
  - Account for uncertain future sectoral inflation rates as in section 3.5, p.24.

§ **Solution:**

- Evaluate robustness of PW to uncertainty in sectoral inflation rates.
- Use inverse robustness, $m(h)$, in eq.(86), p.25:

$$m(h) = -S + \sum_{k=1}^{N} \left[\left(\frac{1 + f_{\text{gen}}(h)}{1 + i_{\text{nom}}} \right)^k (F_0 - M_0) - \left(\frac{1 + f_{\text{it}}(h)}{1 + i_{\text{nom}}} \right)^k T_0 - \left(\frac{1 + f_{\text{e}}(h)}{1 + i_{\text{nom}}} \right)^k P_0 \right] \tag{102}$$

where, if F_0 (period earnings) $\geq M_0$ (period maintenance) (the usual case):

$$f_{\text{gen}}(h) = \begin{cases} \widetilde{f}_{\text{gen}} - s_{\text{gen}} h, & \text{if } h < \dfrac{1 + \widetilde{f}_{\text{gen}}}{s_{\text{gen}}} \\ 0, & \text{else} \end{cases} \tag{103}$$

Or if $F_0 < M_0$ then:

$$f_{\text{gen}}(h) = \widetilde{f}_{\text{gen}} + s_{\text{gen}} h \tag{104}$$

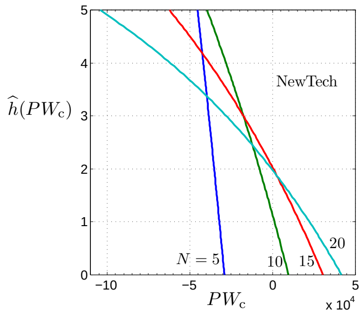

Figure 12: Robustness curves for N = 5, 10, 15 and 20 years, section 3.6, eq.(86). **NewTech.**

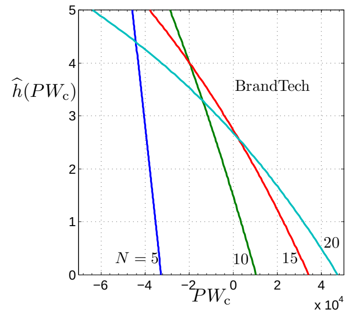

Figure 13: Robustness curves for N = 5, 10, 15 and 20 years, section 3.6, eq.(86). **BrandTech.**

§ **Results.**[13]

- Overview of results in figs. 12 and 13:
  - Fig. 12 has 4 curves from fig. 9 on p.26.
  - NewTech & BrandTech qualitatively similar; different in details. Predicted PW increases with N.
  - Zeroing:
    — No robustness at predicted PW.
    — Like fig. 7, p.23, BrandTech nominally beats NewTech only at large N.
  - Trade off: robustness increases as PW_c decreases.
  - Cost of robustness:
    — Cost of robustness increases as lifetime, N, increases. **Why?**[14]
    — This causes preference reversal among N values for each Tech. **What** dilemma results?

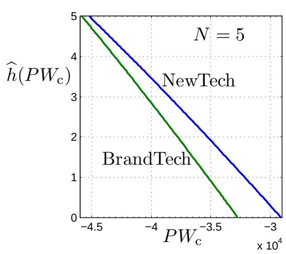

Figure 14:

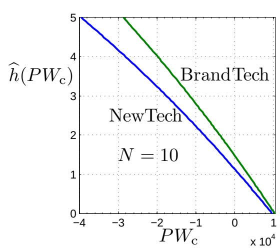

Figure 15:

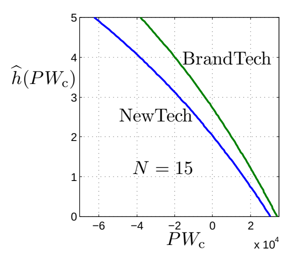

Figure 16:

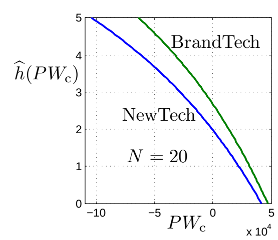

Figure 17:

- Figs. 14–17 combine figs. 12 and 13. Robustness curves for $N = 5, 10, 15, 20$ years, section 3.6, eq.(86). NewTech and BrandTech.
  - Zeroing:
    — NewTech nominally beats BrandTech at $N = 5$ years, but note negative predicted PW.
    — BrandTech and NewTech are nominally very similar at $N = 10$ years.
    — BrandTech nominally beats NewTech at $N = 15$ and 20 years.
  - Trade off and cost of robustness:
    — BrandTech and NewTech are similar.
    — Cost of robustness is large; increases with N. $\frac{\Delta\widehat{h}}{\Delta PW_c}$ (Δrobustness/\$10,000) $\approx$:
    3 at $N = 5$. Low cost of robustness.
    1 at $N = 10$. Medium cost of robustness.
    0.5 at $N = 15$. High cost of robustness.
    0.3 at $N = 20$. High cost of robustness.
  - Choosing a technology and a lifetime:
    — Both Techs have $PW_c < -\$29{,}000$ at $N = 5$. Infeasible.
    — At $N = 10$: $\widehat{h}_{\mathrm{BT}}(0) = 1.5$, $\widehat{h}_{\mathrm{NT}}(0) = 1.0$. Marginally feasible. BT somewhat better.

[13] Calculations with GapZapper application Econ-Dec-Making-Course: Price-Change01
[14] Prediction is always difficult, especially of the future. Robert Storm Petersen, Danish journalist.

— At $N = 15$ and 20: $\widehat{h}_{\rm BT}(0) = 2.8$, $\widehat{h}_{\rm NT}(0) = 2.0$. Both feasible. BT somewhat better.

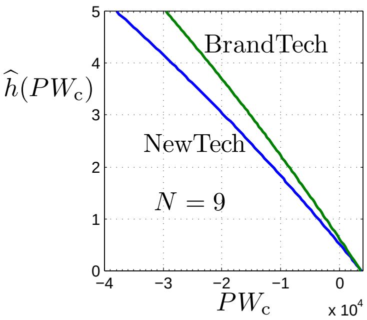

Figure 18:

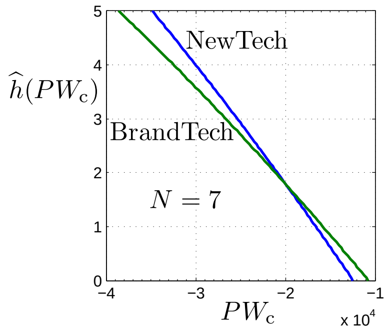

Figure 19:

- Figs. 18 and 19, robustness curves for $N = 7, 9$ years, section 3.6, eq.(86). NewTech and BrandTech.:
  - $N = 9$:
    — NewTech and BrandTech nominally very similar, with $PW_c \approx \$3{,}600$.
    See tables 7, 8, pp.46, 46.
    — BrandTech slightly more robust ($\widehat{h}_{\rm BT} \approx 2.2$ vs $\widehat{h}_{\rm NT} \approx 1.8$) at $PW_c = -\$10{,}000$.
  - $N = 7$:
    — NewTech and BrandTech nominally with $PW_c < -\$10{,}000$.
    — Preference reversal (weak) at $PW_c = -\$20{,}000$.

4 Foreign Exchange Rates

4.1 Basic Concepts

4.1.1 Foreign Exchange Rate Definition

Foreign exchange rate: forex or FX rate.

- The FX rate between two countries is the cost of one currency in the units of the other.
- Example:[15] 1 US$ = 3.8221 NIS: 3.8221 NIS buys 1 US$.
- Usually express FX rate as ratio of "domestic" to "foreign".
- The FX rate, from the Israeli perspective, of the NIS to the US$ is:

$$r_{\text{us}} = \frac{3.8221\text{NIS}}{1\text{US\$}} \tag{105}$$

- "**Strong** NIS against the US$" means **small** r_{us}. NIS gets stronger as r_{us} goes down.
- "**Weak** NIS against the US$" means **large** r_{us}. NIS gets weaker as r_{us} goes up.
- **Question:** For Israelis, is strong FX good or bad?
  - For whom (e.g. importers or exporters; households or producers)?
  - Monetary policy influences the exchange rate. Who's interests should it serve?

[15] 19.10.2012

4.1.2 Purchasing Power Parity (PPP)

- Two exchange rates:
  - NIS–Yen:[16] 1 NIS = 20.7608 Yen: 20.7608 Yen buys 1 NIS.
  - US$–NIS: 1 US$ = 3.8221 NIS: 3.8221 NIS buys 1 US$.
- The NIS FX rates (from the Israeli perspective) for US$ and Yen are:

$$r_{\text{us}} = \frac{3.8221\text{NIS}}{1\text{US\$}} = 3.8821 \text{ NIS per US\$} \tag{106}$$

$$r_{\text{yen}} = \frac{1\text{NIS}}{20.7608\text{Yen}} = 0.048167 \text{ NIS per Yen} \tag{107}$$

- **Question:**
  - We see that $r_{\text{yen}} \ll r_{\text{us}}$.
  - Does this mean that the NIS is stronger against the Yen than against the US$?
- **Answer:** Not necessarily. Depends what 1 Yen buys in Japan, and 1 US$ buys in the US.

§ **Purchasing Power Parity (PPP):**

- Two currencies are at "purchasing power parity" if
  the same amount of any currency (e.g. US$) purchases the same goods in the two countries.
- Example: How many US$ are needed to purchase the same goods in two different countries?
  The 2 countries' currencies are "at parity" if the same amount of US$ are needed.
- Theory: Should be about the same in all countries (**Why?**), unless .... (Truman's 1-armed economist)
- Example: *The Economist's* McDonald's Big Mac Index. The Big Mac costs:[17]
  - US$6.81 in Switzerland. (Buy Swiss Francs with US$, then buy Big Mac).
  - US$4.20 in US.
  - US$4.16 in Japan. (Buy Yen with US$, then buy Big Mac).
  - US$4.13 in Israel. (Buy NIS with US$, then buy Big Mac).
  - US$3.54 in Turkey. (Buy Turkish Lira with US$, then buy Big Mac).
- Swiss Franc is **over-valued** wrt the US$ by 62%:

$$\frac{6.81 - 4.20}{4.20} = 0.62 \tag{108}$$

- Yen is just about **correctly valued** wrt the US$ (undervalued by 0.95%):

$$\frac{4.16 - 4.20}{4.20} = -0.0095 \tag{109}$$

- NIS is just about **correctly valued** wrt the US$ (undervalued by 1.7%):

$$\frac{4.13 - 4.20}{4.20} = -0.017 \tag{110}$$

- Turkish Lira is **under-valued** wrt the US$ by 16%:

$$\frac{3.54 - 4.20}{4.20} = -0.16 \tag{111}$$

§ **Answer to question** following eq.(107) about strong/weak NIS wrt US$ and Yen:

- US$ and Yen are (very nearly) at parity (based on the Big Mac).
- If NIS is undervalued wrt US$ then it is undervalued wrt Yen by same amount.
- NIS is strong or weak to the same degree wrt both the US$ and the Yen.

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[16] 19.10.2012
[17] http://www.economist.com/node/21542808. Accessed 19.10.2012.

4.1.3 Is the Renminbi Weak or Strong with Respect to the US$?

- China opened up to foreign trade and foreign investment in early 1980s.

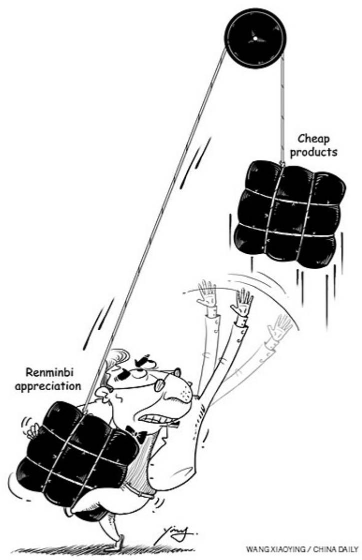

Figure 20: Mark Williams, *China Daily,* 2011-05-05. http://www.chinadaily.com.cn/bizchina/2011-05/05/content_12451717.htm. Accessed 19.10.2012.

- A strong (appreciated) RMB makes Chinese exports expensive for foreigners. Fig. 20.

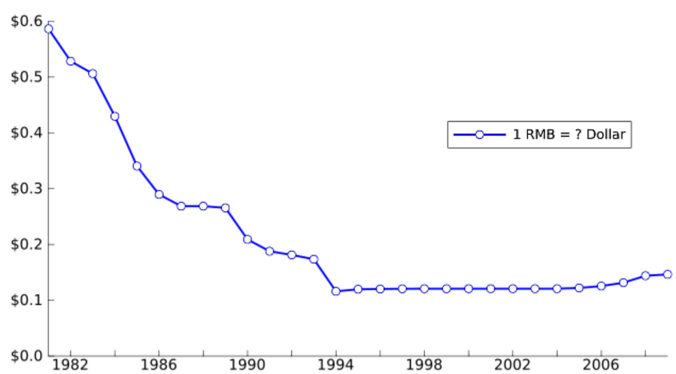

Figure 21: US$ per Renminbi from 1981 to 2009. FX rate from US perspective.
http://en.wikipedia.org/w/index.php?title=File:1_RMB_to_US_dollar.svg&page=1
Accessed 19.10.2012.

- Central Bank of China weakened RMB from '81 to '94 to encourage Chinese exports. Fig. 21.

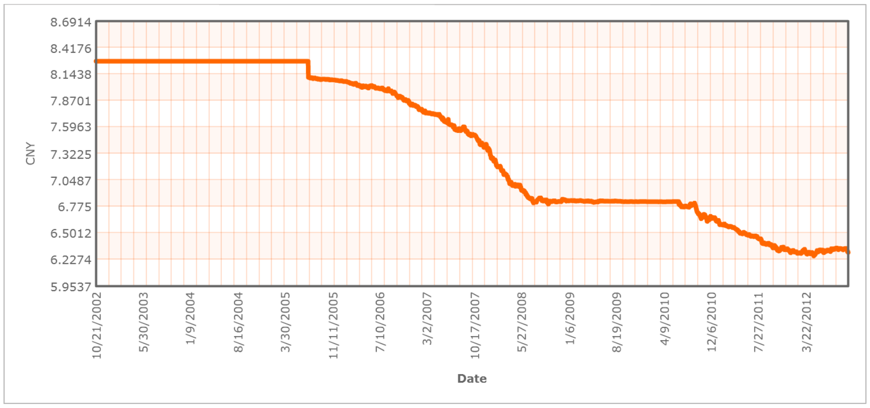

Figure 22: Renminbi (Chinese Yuan, CNY) per US$ from Oct 2002 to Sep 2012. FX rate from Chinese perspective. NY Federal Reserve Bank. http://www.indexmundi.com/xrates/graph.aspx?c1=CNY&c2=USD&days=3650. Accessed 19.10.2012.

- China strengthened RMB against US$ during 2005–2012 to fight inflation. Fig. 22.
- US claims that China keeps RMB weak (depreciated) to boost Chinese exports.
  If true, is this good for US importers (e.g. consumers)? US exporters (e.g. producers)?
- What does the Big Mac Index[18] say:
  - US$4.20 in US.
  - US$2.44 (Buy RMB with US$ then buy Big Mac in China).
  - RMB is **under-valued** wrt the US$ by 42% (compare eqs.(108)–(111), p.32):

$$\frac{2.44 - 4.20}{4.20} = -0.42 \tag{112}$$

- So maybe the Americans are right, the RMB is too weak wrt the US$.
- Or maybe the Big Mac Index doesn't reflect industrial and manufacturing costs in China.
- What is the truth? Hard to say.
- What is certain? The future is **un**certain.

[18] http://www.economist.com/node/21542808. Accessed 19.10.2012

4.1.4 Exchange Rates Can Change Over Time

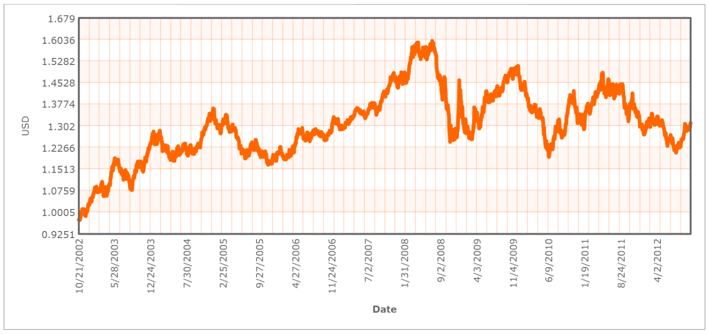

Figure 23: US$ per euro. Dollar-euro exchange rate, 21.10.2002–18.10.2012. Source: NY Federal Reserve Bank. http://www.indexmundi.com/xrates/graph.aspx?c1=USD&c2=EUR&days=3650. Accessed 19.10.2012.

- Fig. 23: US$ per euro, over 10 years. Ranges from 1.0 to 1.6 US$/euro.

Figure 24: NIS per US$ exchange rate. http://www.tradingeconomics.com/israel/currency

- Fig. 24: NIS per US$. Ranges from 1.5 NIS/US$ on introduction (1.1.1986) to 5.0 NIS/US$.

- Why is exchange rate change important? To whom? What are implications for long-term planning?
- Future trends in exchange rate (e.g. fig. 24) may be uncertain. E.g.:
  - FX confidently expected to increase, but:
  - Rate of increase uncertain.
  - Asymmetric uncertainty: $\dfrac{r(t) - \widetilde{r}(t)}{w} \geq h, \quad h \geq 0.$

4.2 Examples.

4.2.1 Foreign Investment without Inflation

§ Based on DeGarmo, pp.390–391.

§ **Background:**

- A US-based firm will invest in a South American country, Thatland.
- The central bank of Thatland devalues its peso by 10% each year wrt the US$.
- The US firm wants a nominal MARR of $i_{\text{dom}} = 0.15$ in its (domestic) currency, the US$.
- There is no inflation in either the US or Thatland.
- For the values below, **is the investment justified economically?**

§ **Cash flow:**

- r_k = exchange rate, US$/peso, at year k, $k = 0, \ldots, N$, where:

$$r_k = \frac{r_0}{(1+\varepsilon)^k}, \quad r_0 = 0.01, \quad \varepsilon = 0.1 \tag{113}$$

  r_k small implies weak peso and strong US$.
- $N = 5$ years, project duration.
- S_{for} = 50,000,000 peso initial investment in Thatland.
- S_{dom} = initial investment in domestic currency, US$:

$$S_{\text{dom}} = r_0 S_{\text{for}} \tag{114}$$

- $R_{k,\text{for}}$ = net revenue (nominal values; there is no inflation) in peso in year k. Anticipated values:

$$R_{k,\text{for}} = 20,000,000 \text{ peso for } k = 1,2,3 \tag{115}$$
$$R_{k,\text{for}} = 30,000,000 \text{ peso for } k = 4,5 \tag{116}$$

- $R_{k,\text{dom}}$ = net revenue (nominal values; there is no inflation) in US$ for year k:

$$R_{k,\text{dom}} = r_k R_{k,\text{for}} \tag{117}$$

  Revenue remittances from Thatland to the US are made each year.
- i_{dom} = firm's nominal MARR = 0.15.

§ **Solution:** Calculate the PW in the US firm's domestic currency:

$$\begin{aligned}
\text{PW}_{\text{dom}} &= -S_{\text{dom}} + \sum_{k=1}^{N} (1+i_{\text{dom}})^{-k} R_{k,\text{dom}} & (118)\\
&= -r_0 S_{\text{for}} + \sum_{k=1}^{N} (1+i_{\text{dom}})^{-k} r_k R_{k,\text{for}} & (119)\\
&= -r_0 S_{\text{for}} + \sum_{k=1}^{N} (1+i_{\text{dom}})^{-k} \frac{r_0}{(1+\varepsilon)^k} R_{k,\text{for}} & (120)\\
&= -r_0 S_{\text{for}} + \sum_{k=1}^{N} \frac{r_0}{[(1+i_{\text{dom}})(1+\varepsilon)]^k} R_{k,\text{for}} & (121)\\
&= \$91{,}652.37 & (122)
\end{aligned}$$

- The domestic PW is positive at the firm's MARR so the project is economically justified.

4.2.2 Foreign Investment with Inflation.

§ Extension of section 4.2.1, p.36.

§ **Background:** Same as section 4.2.1 except with inflation in US and Thatland.

§ **Inflation and interest rates:**

- f_{for} = constant inflation rate in foreign currency, the peso.
- f_{dom} = constant inflation rate in domestic currency, the US$.
- $i_{\text{nom,dom}}$ = nominal domestic MARR: 0.15 interest rate.
- $i_{\text{r,dom}}$ = real domestic interest rate, eq.(32), p.11:

$$i_{\text{r,dom}} = \frac{i_{\text{nom,dom}} - f_{\text{dom}}}{1 + f_{\text{dom}}} \tag{123}$$

§ **Real and nominal cash flows:**

- S_{for} = initial investment in foreign currency, peso.
- S_{dom} = initial investment in domestic currency, US$, eq.(114), p.36:

$$S_{\text{dom}} = r_0 S_{\text{for}} \tag{124}$$

- $R_{k,\text{for}}$ = real revenue in foreign currency, peso, in year k, eqs.(115) and (116), p.36:

$$R_{k,\text{for}} = 20,000,000 \text{ peso for } k = 1,2,3 \tag{125}$$

$$R_{k,\text{for}} = 30,000,000 \text{ peso for } k = 4,5 \tag{126}$$

  Peso revenue in base-year ($k = 0$) pesos.

- $A_{k,\text{for}}$ = nominal revenue in foreign currency, peso, in year k:

$$A_{k,\text{for}} = (1 + f_{\text{for}})^k R_{k,\text{for}} \tag{127}$$

  Peso revenue in current-year actual peso currency (bills or bank balance).

- $A_{k,\text{dom}}$ = nominal revenue in domestic currency, US$, in year k:

$$A_{k,\text{dom}} = r_k A_{k,\text{for}} \tag{128}$$

  r_k is exchange rate, eq.(113), p.36:

$$r_k = \frac{r_0}{(1+\varepsilon)^k}, \quad r_0 = 0.01, \quad \varepsilon = 0.1 \tag{129}$$

  $A_{k,\text{dom}}$ is US$ revenue in current-year actual US$ currency (bills or bank balance).

- $R_{k,\text{dom}}$ = real revenue in domestic currency, US$, in year k:

$$R_{k,\text{dom}} = (1 + f_{\text{dom}})^{-k} A_{k,\text{dom}} \tag{130}$$

  US$ revenue in base-year ($k = 0$) US$.

§ **Question:**

Is the project economically justified, accounting for changing exchange rate and inflation?

§ **Solution:** Calculate PW in domestic currency, US$.

- PW is the same in real and nominal dollars, because PW is in base-year US$.
  - We first evaluate the PW starting with nominal domestic revenue.
  - We then evaluate the PW starting with real domestic revenue.

  ◦ These results will be the same.

- The domestic PW, **starting in domestic nominal US$:**

$$\text{PW}_{\text{dom}} = -S_{\text{dom}} + \sum_{k=1}^{N}(1+i_{\text{nom,dom}})^{-k}A_{k,\text{dom}} \tag{131}$$

Use the exchange rate to change domestic to foreign nominal currency:

$$= -r_0 S_{\text{for}} + \sum_{k=1}^{N}(1+i_{\text{nom,dom}})^{-k}r_k A_{k,\text{for}} \tag{132}$$

Introduce the devaluation rate, ε:

$$= -r_0 S_{\text{for}} + \sum_{k=1}^{N}(1+i_{\text{nom,dom}})^{-k}r_0(1+\varepsilon)^{-k} A_{k,\text{for}} \tag{133}$$

Transform nominal to real foreign currency with inflation rate:

$$= -r_0 S_{\text{for}} + \sum_{k=1}^{N}(1+i_{\text{nom,dom}})^{-k}r_0(1+\varepsilon)^{-k}(1+f_{\text{for}})^k R_{k,\text{for}} \tag{134}$$

$$= -r_0 S_{\text{for}} + r_0\sum_{k=1}^{N}(1+i_{\text{nom,dom}})^{-k}\left(\frac{1+f_{\text{for}}}{1+\varepsilon}\right)^k R_{k,\text{for}} \tag{135}$$

  ◦ Eq.(135) is the same as eq.(121), p.36, if $f_{\text{for}} = 0$.
  ◦ Eq.(122) was $\text{PW}_{\text{dom}} = \$91,652.37$ as in solid blue curve ($\varepsilon = 0.10$) in fig. 25, p.39, at $f_{\text{for}} = 0$.
  ◦ The domestic PW in eq.(135):
    — Does not depend on domestic inflation.
    — Does depend on foreign inflation.

  Explanation:
    — Real revenue, $R_{k,\text{for}}$, inflates in Thatland.
    — Remittances to US are in nominal US currency, $A_{k,\text{dom}}$, through the exchange rate.
    — Domestic inflation is circumvented.
    — The foreign operator pays for foreign inflation.
  ◦ The domestic PW in eq.(135):
    — Depends on foreign inflation rate, f_{for}, and peso devaluation rate, ε. Fig. 25, p.39.
    — These two effects tend to counteract each other.

  Explanation:
    — As **foreign inflation increases:** the foreign nominal revenues, eq.(127) p.37, increase:

$$A_{k,\text{for}} = (1+f_{\text{for}})^k R_{k,\text{for}} \tag{136}$$

This increases domestic nominal revenue, eq.(128):

$$A_{k,\text{dom}} = r_k A_{k,\text{for}} \tag{137}$$

and **increases the domestic PW,** eq.(131):

$$\text{PW}_{\text{dom}} = -S_{\text{dom}} + \sum_{k=1}^{N}(1+i_{\text{nom,dom}})^{-k}A_{k,\text{dom}} \tag{138}$$

    — As the **peso devalues** (ε increases): the US$ strengthens (r_k in eq.(113) falls) and the domestic real revenue decreases, eq.(117):

$$R_{k,\text{dom}} = r_k R_{k,\text{for}} \tag{139}$$

which **decreases the domestic PW.**

- The domestic PW, **starting in domestic real US$:**

$$\text{PW}_{\text{dom}} = -S_{\text{dom}} + \sum_{k=1}^{N}(1+i_{\text{r,dom}})^{-k}R_{k,\text{dom}} \tag{140}$$

Using eqs.(123) and (130):

$$= -S_{\rm dom} + \sum_{k=1}^{N} \left(1 + \frac{i_{\rm nom,dom} - f_{\rm dom}}{1+f_{\rm dom}}\right)^{-k} (1+f_{\rm dom})^{-k} A_{k,\rm dom} \quad (141)$$

$$= -S_{\rm dom} + \sum_{k=1}^{N} \left(\frac{1 + i_{\rm nom,dom}}{1+f_{\rm dom}}\right)^{-k} (1+f_{\rm dom})^{-k} A_{k,\rm dom} \quad (142)$$

$$= -S_{\rm dom} + \sum_{k=1}^{N} (1 + i_{\rm nom,dom})^{-k} A_{k,\rm dom} \quad (143)$$

which is precisely eq.(131).

PW is the same in real and nominal dollars, because PW is in base-year US$. QED

§ **Results,** fig. 25, p.39.[19] PW ≥ 0 and the project is economically justified, for:

- $\varepsilon = 0.05$ or 0.10 and $-0.06 \leq f_{\rm for} \leq 0.06$. Green and Blue curves positive.
- $\varepsilon = 0.15$ and $-0.02 \leq f_{\rm for}$.
- Devaluation, ε, between 5% and 15% has substantial impact of PW.
- Foreign inflation, $f_{\rm for}$,, between −6% and +6% has substantial impact of PW.

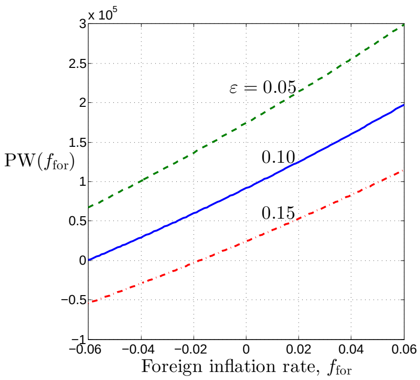

Figure 25: Present worth vs foreign inflation, section 4.2.2, eq.(135).

4.2.3 Foreign Investment with Inflation: Two Design Alternatives

§ Extension of section 4.2.2, p.37.

§ **Background.**

- The US firm is considering two design alternatives:
  - Both designs have the same incomes per year, but their operating costs vary differently.
  - Design 1:
    - — Same as section 4.2.2: fairly uniform costs throughout life.
    - — Net real revenue in foreign currency: eqs.(115) and (116), p.36:

$$R_{k,\rm for} = 20,000,000 \text{ peso for } k = 1,2,3 \quad (144)$$

$$R_{k,\rm for} = 30,000,000 \text{ peso for } k = 4,5 \quad (145)$$

    - — Initial peso capital investment: $S_{\rm for} = 50{,}000{,}000$ peso.
  - Design 2:
    - — Low early costs; high early net revenue. Large late costs; low late net revenue.
    - — Net real revenue in foreign currency:

$$R_{1,\rm for} = 50,000,000 \text{ peso} \quad (146)$$

[19] Calculated with GapZapper application Econ-Dec-Making-Course: Price-Change02.

$$R_{2,\text{for}} = 40,000,000 \text{ peso} \tag{147}$$

$$R_{k,\text{for}} = 10,000,000 \text{ peso for } k = 3, 4, 5 \tag{148}$$

— Initial peso capital investment: S_{for} = 67,289,184 peso.

§ **Inflation and interest rates:** same as section 4.2.2, p.37:

§ **Inflation and interest rates:**

- f_{for} = constant inflation rate in foreign currency, the peso.
- f_{dom} = constant inflation rate in domestic currency, the US$.
- $i_{\text{nom,dom}}$ = nominal domestic MARR: 0.15 interest rate.
- $i_{\text{r,dom}}$ = real domestic interest rate, eq.(32), p.11:

$$i_{\text{r,dom}} = \frac{i_{\text{nom,dom}} - f_{\text{dom}}}{1 + f_{\text{dom}}} \tag{149}$$

§ **Question:**

Which design is preferable, as a function of devaluation (pichut) rate ε and foreign inflation rate f_{for}?

§ **Solution:** Calculate the PW in domestic currency, US$, eq.(135), p.38:

$$\text{PW}_{\text{dom}} = -r_0 S_{\text{for}} + r_0 \sum_{k=1}^{N} (1 + i_{\text{nom,dom}})^{-k} \left(\frac{1 + f_{\text{for}}}{1 + \varepsilon} \right)^k R_{k,\text{for}} \tag{150}$$

§ **Results,** fig. 26.[20]

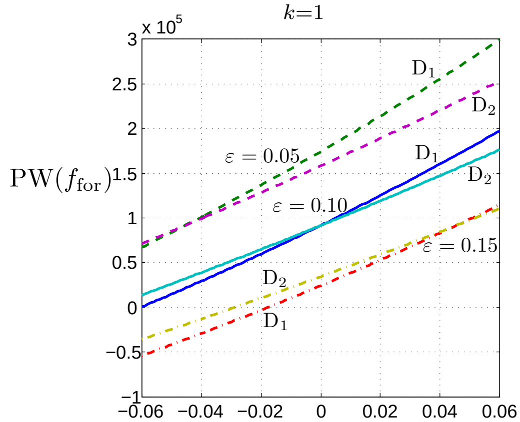

Figure 26: Present worth vs foreign inflation, section 4.2.3, eq.(150) for two designs.

- D_1 and D_2 have same PW at $\varepsilon = 0.1$ and $f_{\text{for}} = 0$: intersection of solid lines in fig. 26.
- From fig. 26: PW(D_1) > PW(D_2) so D_1 preferred over D_2 at:

$$\varepsilon = 0.05: \quad \text{for } f_{\text{for}} > -0.048 \tag{151}$$

$$\varepsilon = 0.10: \quad \text{for } f_{\text{for}} > 0 \tag{152}$$

$$\varepsilon = 0.15: \quad \text{for } f_{\text{for}} > 0.048 \tag{153}$$

- D_2 **improves** wrt D_1 as rate of devaluation (ε) increases. Explanation:
  - r_k = exchange rate, US$/peso, at year k, $k = 0, \ldots, N$, where:

$$r_k = \frac{r_0}{(1+\varepsilon)^k}, \quad r_0 = 0.01, \quad \varepsilon = 0.1 \tag{154}$$

  - Foreign devaluation (large ε) reduces domestic PW, see eqs.(154), (150), because remittances from foreign to domestic currency go through FX.
  - D_2 has large revenues early, before devaluations accumulate.
- D_2 **deteriorates** wrt D_1 as foreign inflation (f_{for}) increases. Explanation:
  - Foreign inflation (large f_{for}) increases domestic PW, see eq.(150), because foreign inflation increases the foreign nominal revenues that go through FX.
  - D_2 has large revenues early, before inflation accumulates.

[20] Calculated with GapZapper application Econ-Dec-Making-Course: Price-Change02.

4.2.4 Foreign Investment with Uncertain Inflation and Uncertain Devaluation: Two Design Alternatives.

§ Extension of section 4.2.3, p.39.

§ **Background:**

- Choose between two design alternatives in section 4.2.3.
- Foreign inflation, $f_{\rm for}$, and devaluation rate, ε, are constant but of uncertain values.

§ **Uncertainty model:**

- $\widetilde{f}_{\rm for}$ = estimated rate of foreign inflation.
- s_f = error estimate for $\widetilde{f}_{\rm for}$. $s_f = 0.6\widetilde{f}_{\rm for}$. **Question:** What does this mean?
- $\widetilde{\varepsilon}$ = estimated rate of foreign devaluation.
- s_ε = error estimate for $\widetilde{\varepsilon}$. $s_\varepsilon = 0.3\widetilde{\varepsilon}$.

$$\mathcal{U}(h) = \left\{ \varepsilon, f_{\rm for} : \ \left| \frac{\varepsilon - \widetilde{\varepsilon}}{s_\varepsilon} \right| \le h, \ \left| \frac{f_{\rm for} - \widetilde{f}_{\rm for}}{s_f} \right| \le h \right\}, \quad h \ge 0 \tag{155}$$

We are ignoring $f > -1$ for simplicity.

§ **System model:** PW, eq.(150), p.40:

$$\mathrm{PW}_{\rm dom} = -r_0 S_{\rm for} + r_0 \sum_{k=1}^{N} (1 + i_{\rm nom,dom})^{-k} \left(\frac{1 + f_{\rm for}}{1 + \varepsilon} \right)^k R_{k,\rm for} \tag{156}$$

§ **Performance requirement:**

$$\mathrm{PW}_{\rm dom} \ge PW_{\rm c} \tag{157}$$

§ **Robustness:** greatest tolerable uncertainty:

$$\widehat{h}(PW_{\rm c}) = \max \left\{ h : \ \left(\min_{\varepsilon, f \in \mathcal{U}(h)} \mathrm{PW}_{\rm dom} \right) \ge PW_{\rm c} \right\} \tag{158}$$

◦ Inner minimum, $m(h)$, occurs at (assuming $R_{k,\rm for} \ge 0$):

$$\varepsilon = \widetilde{\varepsilon} + s_\varepsilon h, \quad f_{\rm for} = \widetilde{f}_{\rm for} - s_f h \tag{159}$$

Explanation:

— Algebraic: see eq.(156).

— Economic:

◦ Lowered foreign inflation ($f_{\rm for}$ down) reduces nominal foreign revenue (bad).

◦ Raised foreign devaluation (ε up) decreases exchange value of foreign revenue (bad).

◦ Inverse robustness is:

$$m(h) = -r_0 S_{\rm for} + r_0 \sum_{k=1}^{N} (1 + i_{\rm nom,dom})^{-k} \left(\frac{1 + \widetilde{f}_{\rm for} - s_f h}{1 + \widetilde{\varepsilon} + s_\varepsilon h} \right)^k R_{k,\rm for} \tag{160}$$

◦ Two properties of $m(h)$:

— $m(0)$ = predicted putative PW. **Why?** What does "zeroing" mean?

— $m(h)$ decreases as h increases. **Why?** What does this "trade off" mean?

§ **Results,** fig. 27.[21]

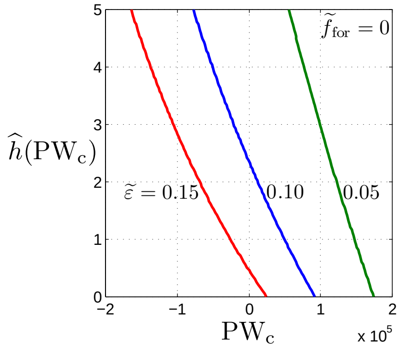

Figure 27: Robustness curves for section 4.2.4, eq.(160), for design 1 from section 4.2.3.

- Robustness curves for design 1 of section 4.2.3 in fig. 27.
  - ○ Zeroing: horizontal intercepts equal PW values for D_1 in fig. 26 at $f_{\mathrm{for}} = 0$.
  - ○ Trade off and cost of robustness: roughly the same for all three ε values.
  - ○ Cost of robustness quite substantial: Decrease PW_c by \$$10^5$ to raise $\widehat{h}$ by ∼2.5.

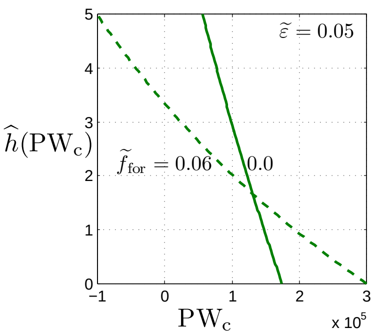

Figure 28: Robustness curves for section 4.2.4, eq.(160), for design 1 from section 4.2.3. $\widetilde{\varepsilon} = 0.05$.

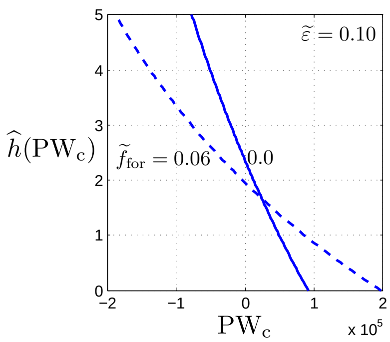

Figure 29: Robustness curves for section 4.2.4, eq.(160), for design 1 from section 4.2.3. $\widetilde{\varepsilon} = 0.10$.

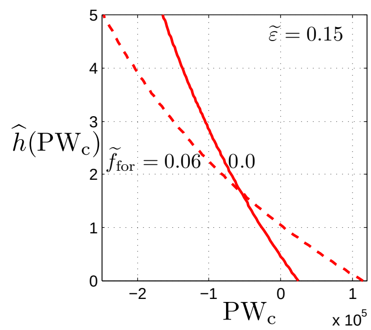

Figure 30: Robustness curves for section 4.2.4, eq.(160), for design 1 from section 4.2.3. $\widetilde{\varepsilon} = 0.15$.

- Comparing inflation levels, figs. 28–30:
  - ○ Relations to fig. 27:
  - — Solid green curves in figs. 27 and 28 are the same.
  - — Solid blue curves in figs. 27 and 29 are the same.
  - — Solid red curves in figs. 27 and 30 are the same.
- Preference reversals in all three figures:
  - ○ $\widetilde{f}_{\mathrm{for}} = 0.06 \succ \widetilde{f}_{\mathrm{for}} = 0.0$ at large PW_c.
  - ○ $\widetilde{f}_{\mathrm{for}} = 0.0 \succ \widetilde{f}_{\mathrm{for}} = 0.06$ at small PW_c.
- Reason for preference reversals:
  - ○ Foreign inflation **improves domestic PW:** robustness curve shifts rt. with positive $\widetilde{f}_{\mathrm{for}}$.
  - ○ Foreign inflation **increases cost of robustness:** slope less negative with positive $\widetilde{f}_{\mathrm{for}}$.

[21] Calculated with GapZapper application Econ-Dec-Making-Course: Price-Change03.

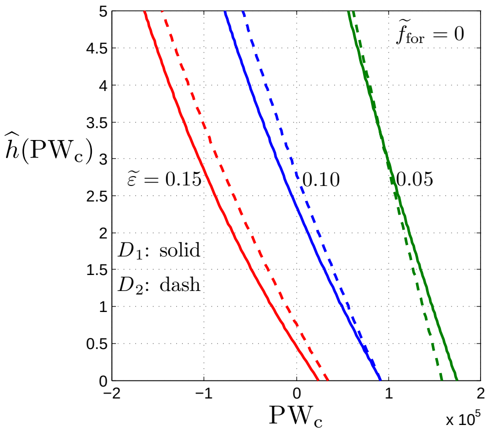

Figure 31: Robustness curves for section 4.2.4, eq.(160), for designs 1 and 2 from section 4.2.3. $\widetilde{f}_{\rm for} = 0$.

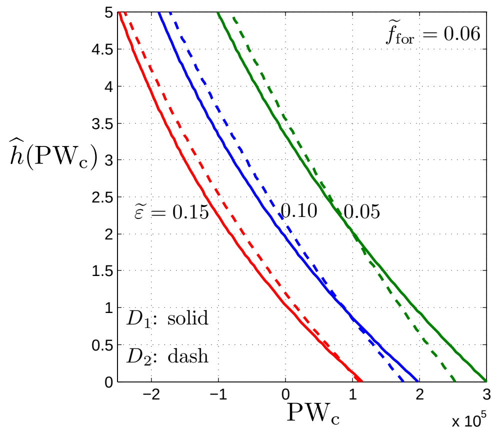

Figure 32: Robustness curves for section 4.2.4, eq.(160), for designs 1 and 2 from section 4.2.3. $\widetilde{f}_{\rm for} = 0.06$.

- Comparing Designs 1 and 2, figs. 31 and 32:
  - ◦ Quite similar robustness curves.
  - ◦ Some curve-crossing and preference reversal.
  - ◦ Cost of robustness much greater at $\widetilde{f}_{\rm for} = 0.06$ than at $\widetilde{f}_{\rm for} = 0$.

A CPI and PPI Data

Year	CPI[a]	CPI[b]
1983	0.5083	0.5789
1984	2.1842	2.4241
1985	10.4941	11.9737
1986	16.5730	19.0538
1987	19.8071	22.8974
1988	23.0014	26.6526
1989	27.6294	31.5755
1990	32.1383	35.7658
1991	38.3555	41.9481
1992	43.0233	46.6649
1993	47.8500	50.8781
1994	53.8109	55.4277
1995	59.0044	60.5825
1996	66.6024	66.9623
1997	72.2287	72.4745
1998	75.2583	75.9451
1999	79.6770	80.7819
2000	81.3496	83.1488
2001	81.9451	83.1983
2002	87.3649	86.6649
2003	87.0853	88.7548
2004	87.0853	89.2718
2005	87.3445	90.0473
2006	90.3683	93.1494
2007	89.7668	93.1287
2008	94.0714	98.2871
2009	97.4552	98.5423
2010	99.8115	100.0000
2011	104.0000	103.5000
2012	105.0000	103.9000

Table 5: Data for fig 1. Israel CBS.
[a] June Consumer price index—general, fig. 1, solid. Average 2010 = 100.
[b] June Consumer price index—without vegetables, fruits, or housing fig. 1, dash. Average 2010 = 100.

Year	PPI^a	PPI^b
1995	62.0882	53.5620
1996	67.5042	58.2342
1997	71.0776	61.3169
1998	73.4785	63.3881
1999	78.9503	68.1085
2000	83.2496	71.8173
2001	83.3054	71.8655
2002	86.2088	74.3702
2003	88.5539	76.3932
2004	94.5840	81.5953
2005	99.3858	85.7377
2006	107.7000	92.9101
2007	108.4000	93.5140
2008	124.5000	107.4030
2009	111.7000	96.3608
2010	115.7000	99.8115
2011	125.8000	108.5245
2012	129.1000	111.3713

Table 6: Data for fig 1. Israel CBS.
[a] June Producer Price Index—Manufacturing output for domestic market, fig. 1, dot-dash. The PPI data are for Average 2005 = 100.
[b] Same as column 2 except that the data have been adjusted to year-2005 basis: $\text{PPI}^b = \text{PPI}^a \times \text{CPI}_{\text{gen}}(2010)/\text{PPI}(2005)$. $\text{CPI}_{\text{gen}}(2010)$ is taken from table 6 and is very nearly 100.

B Matlab Code for Figure 3

```
A1=35000;
r=0.06;
wtf=0.08;
s=0.25*wtf;
h=linspace(0,4,100);
rho=(1+r)./(1 + wtf + s*h);
m=A1*(rho.^4 - 1)./(rho-1);
plot(mh,h)
```

C Data for Section 3.2, p.16

Year	PW ($)
5	−29105.95587478168
6	−19362.38431177076
7	−10731.04682861352
8	−3088.363635506801
9	3675.75041628953
10	9659.342290449044
11	14949.72130610995
12	19624.62035638528
13	23753.23263991057
14	27397.13716783587
15	30611.12490218040
16	33443.93612363012
17	35938.91850200194
18	38134.61433670044
19	40065.28453500854
20	41761.37609175674

Table 7: NewTech data for section 3.2, p.16.

D Data for Section 3.4, p.22

Year	PW ($)
5	−32738.98720804903
6	−21995.20174743337
7	−12439.36644182916
8	−3942.256868031880
9	3611.447558470129
10	10324.64824251388
11	16289.15439112525
12	21586.86899829903
13	26290.84875341062
14	30466.25121977853
15	34171.18122140647
16	37457.44711573812
17	40371.23650276914
18	42953.71991213277
19	45241.59010731967
20	47267.54383883678

Table 8: BrandTech data for section 3.4, p.22.