14. **Investment with uncertain costs and returns, revisited.** (p.22) Return to problem 6 and consider uncertainty in the operating costs and yearly returns. The initial cost is \$640,000. The income is estimated to be \$180,000 at the end of each year, but this estimate may err by 30% or more. The annual cost of operating the new system is estimated to be \$44,000 at the end of each of the first two years, and to decrease by \$4,000 in each of the subsequent years. The operating costs may err by 10% or more. The company's minimal acceptable rate of return (MARR) is 15%. The system has no salvage value at the end of 8 years. Derive the robustness function of the PW.

Solution to Problem 14, Investment with uncertain costs and returns, revisited, (p.5).

- S = Initial cost of the project = \$640,000.
- $\widetilde{R}_k$ = estimated revenue at the end of kth period = \$180,000.
- $\widetilde{C}_k$ = estimated operating cost at the end of kth period.
  = \$44,000 for $k = 1, 2$.
  = $\$44,000 - 4,000(k-2)$ for $k = 3, \ldots, 8$.
- N = number of periods = 8.
- MARR = 15%, so $i = 0.15$.

The PW is:

$$PW(R, C) = -S + \sum_{k=1}^{N}(1+i)^{-k}R_k - \sum_{k=1}^{N}(1+i)^{-k}C_k \tag{94}$$

The info-gap model for cost and return uncertainty:

$$\mathcal{U}(h)\left\{R, C: \ \left|\frac{R_k - \widetilde{R}_k}{\varepsilon_R \widetilde{R}_k}\right| \le h, \ \left|\frac{C_k - \widetilde{C}_k}{\varepsilon_C \widetilde{C}_k}\right| \le h, \ k = 1, \ldots, N\right\}, \quad h \ge 0 \tag{95}$$

where $\varepsilon_R = 0.3$ and $\varepsilon_C = 0.1$.

The robustness is defined as:

$$\widehat{h} = \max\left\{h: \left(\min_{R,C\in\mathcal{U}(h)} \mathrm{PW}(R, C)\right) \ge \mathrm{PW_c}\right\} \tag{96}$$

Let $m(h)$ denote the inner minimum, which occurs when:

$$R_k = \widetilde{R}_k - \varepsilon_R\widetilde{R}_k h = \widetilde{R}_k(1 - \varepsilon_R h), \quad C_k = \widetilde{C}_k + \varepsilon_C\widetilde{C}_k h = \widetilde{C}_k(1 + \varepsilon_C h) \tag{97}$$

Thus:

$$m(h) = -S + \sum_{k=1}^{N}(1+i)^{-k}\widetilde{R}_k(1 - \varepsilon_R h) - \sum_{k=1}^{N}(1+i)^{-k}\widetilde{C}_k(1 + \varepsilon_C h) \tag{98}$$

$$= \mathrm{PW}(\widetilde{R}, \widetilde{C}) - h \underbrace{\sum_{k=1}^{N}(1+i)^{-k}(\widetilde{R}_k\varepsilon_R + \widetilde{C}_k\varepsilon_C)}_{Q} \tag{99}$$

Equate to $\mathrm{PW_c}$ and solve for h to obtain the robustness:

$$\widehat{h} = \frac{\mathrm{PW}(\widetilde{R}, \widetilde{C}) - \mathrm{PW_c}}{Q} \tag{100}$$

or zero if this is negative.